Machine Learning and Pervasive Computing

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Overview and Structure

Outline

Unsupervised learning

Self Organizing Maps

- Introduction
- Definition
- Example

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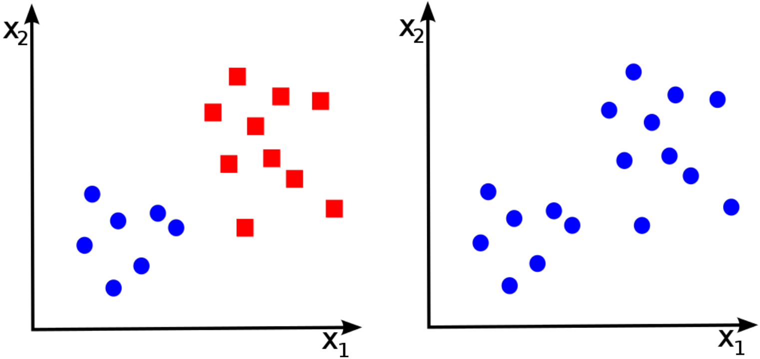
Unsupervised learning



Supervised:
$\{(x_{1,1}, x_{1,2}) \rightarrow y_1, (x_{2,1}, x_{2,2}) \rightarrow y_2, \ldots, (x_{n,1}, x_{n,2}) \rightarrow y_n\}$
Unsupervised: $\{(x_{1,1}, x_{1,2}), (x_{2,1}, x_{2,2}), \ldots, (x_{n,1}, x_{n,2})\}$

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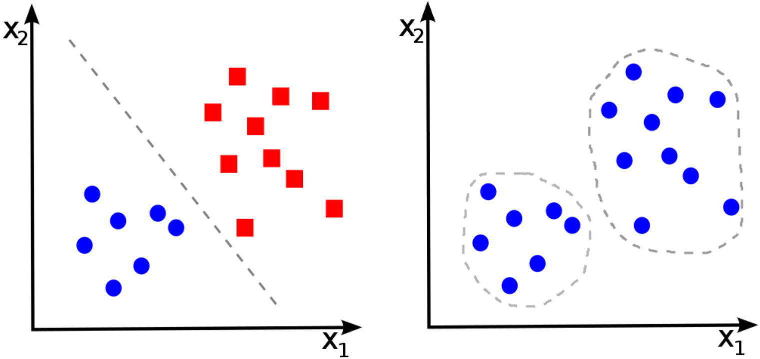
Unsupervised learning



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Unsupervised learning

k-means algorithm

Iteratively find k clusters in the data

Init Randomly choose k points as initial cluster centroids

Repeat :

- → Assign samples to these cluster centroids conditioned on distance
- → Move cluster centroids to the center weight of the points associated to them

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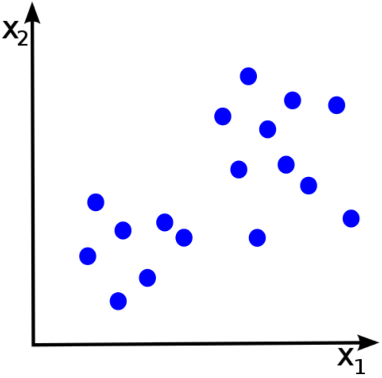
Unsupervised learning

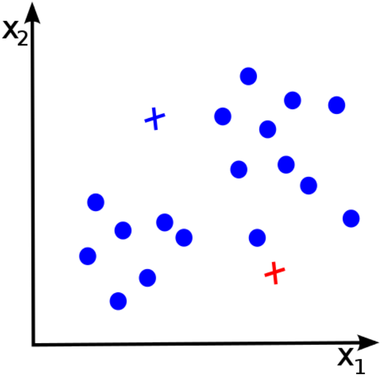
k-means algorithm



Unsupervised learning

k-means algorithm

Init: k cluster centroids μ_i chosen randomly



Unsupervised learning

k-means algorithm

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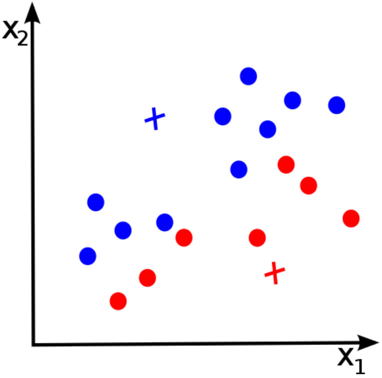


Unsupervised learning

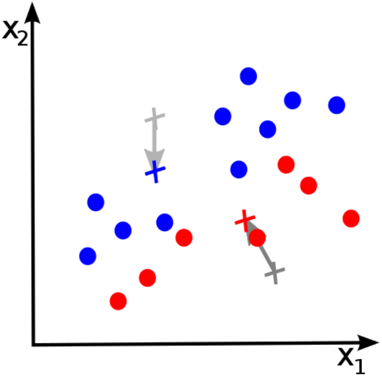
k-means algorithm

Init: k cluster centroids μ_i chosen randomly

Repeat:

1: assign samples to centroids conditioned on distance

2: $\mu_j(t+1) = \frac{1}{C_j} \sum_{i=1}^{C_j} x^{(i)}$



Unsupervised learning

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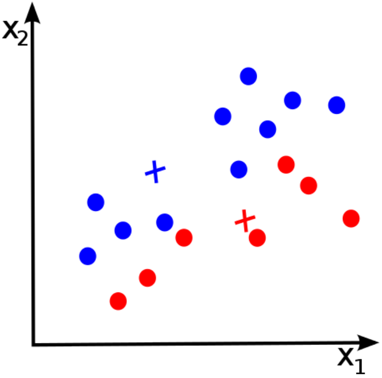

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Unsupervised learning

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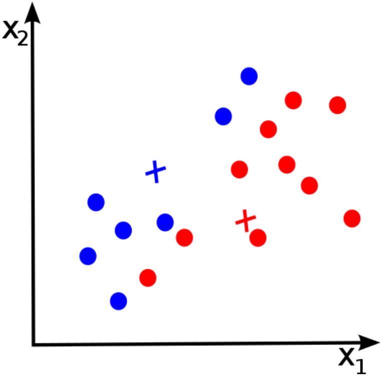

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Unsupervised learning

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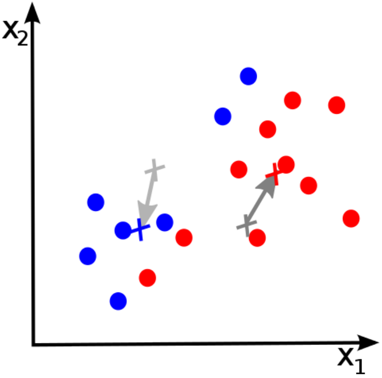

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Unsupervised learning

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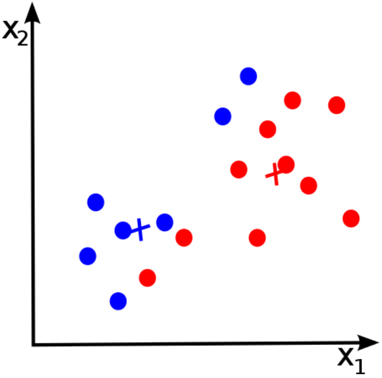

Unsupervised learning

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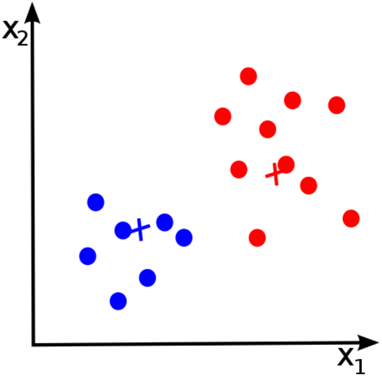

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Unsupervised learning

k-means algorithm

How to randomly initialise the k-means algorithm

The k-means algorithms may compute different solutions for different initial choice of cluster centroids

With respect to the overall distance of the samples to their cluster centroids, k-means might run into local optima

Unsupervised learning

k-means algorithm

How to randomly initialise the k-means algorithm

The k-means algorithms may compute different solutions for different initial choice of cluster centroids

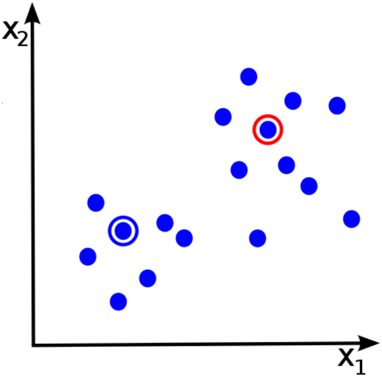
With respect to the overall distance of the samples to their cluster centroids, k-means might run into local optima

Common choice of the initial k cluster centroids

Choose the initial k cluster centroids randomly from the set of training samples

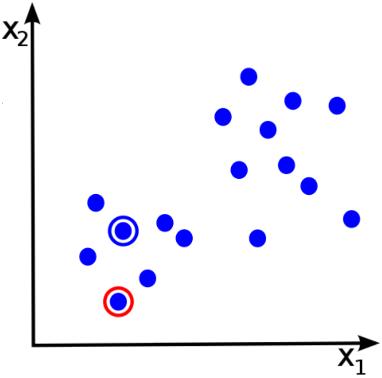
Unsupervised learning

k-means algorithm



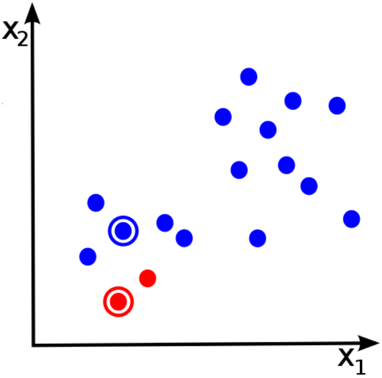
Unsupervised learning

k-means algorithm



Unsupervised learning

k-means algorithm



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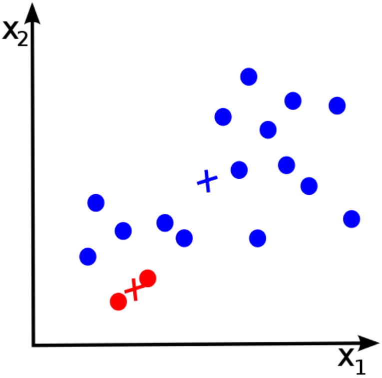
Unsupervised learning

k-means algorithm



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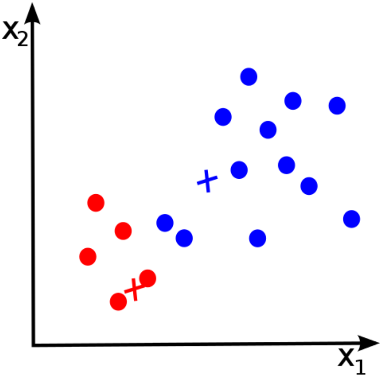
Unsupervised learning

k-means algorithm



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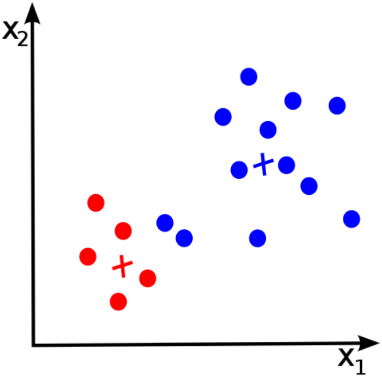
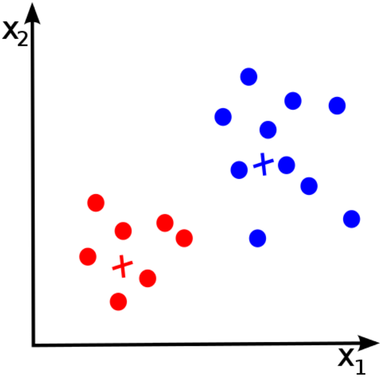
Unsupervised learning

k-means algorithm



Unsupervised learning

k-means algorithm



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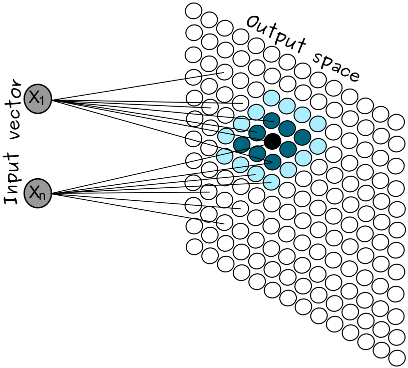
Outline

Introduction to self organising maps (SOM)

Proposed by Teuvo Kohonen[1]

→ Model of the self-organisation of neural connections

→ Maps high dimensional input to low dimensional (e.g. 2D) output



[1] Teuvo Kohonen, *Self-Organizing Maps*, Springer, 2001.

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Introduction to self organising maps (SOM)

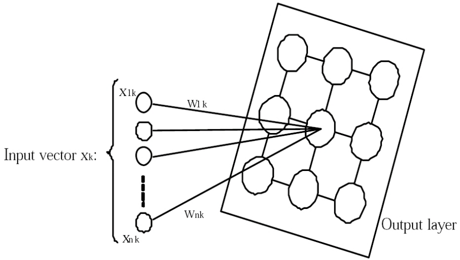
Relation to Neural Networks

Similarity

- Weighted inputs mapped to vector of outputs

Difference

- Considers neighbourhood relation and ordering of output layer
- Unsupervised
- Alternative learning and updating



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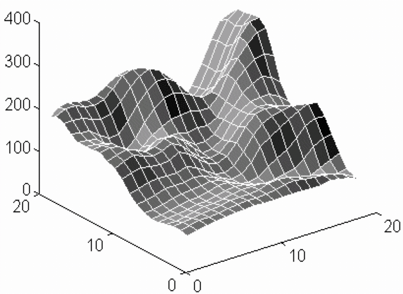
Introduction to self organising maps

Represent all points in a source space by points in a target space

Given a sequence of points in a sample space,

Create a mapping of these points into a target space that respects the neighbourhood relation in the sample space



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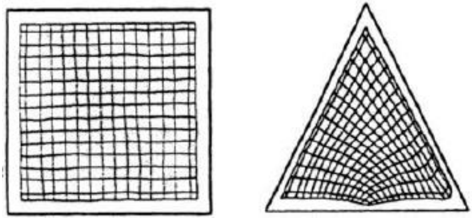
Introduction to self organising maps



SOM is a topology preserving lattice of predefined number of nodes

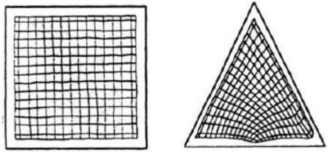
Represents topology of elements in input space.

Algorithm inherits self-organisation property

- Able to produce organisation starting from total disorder.
- Defines and preserves neighbourhood structure between nodes

Learning by two layer neural network

Introduction to self organising maps



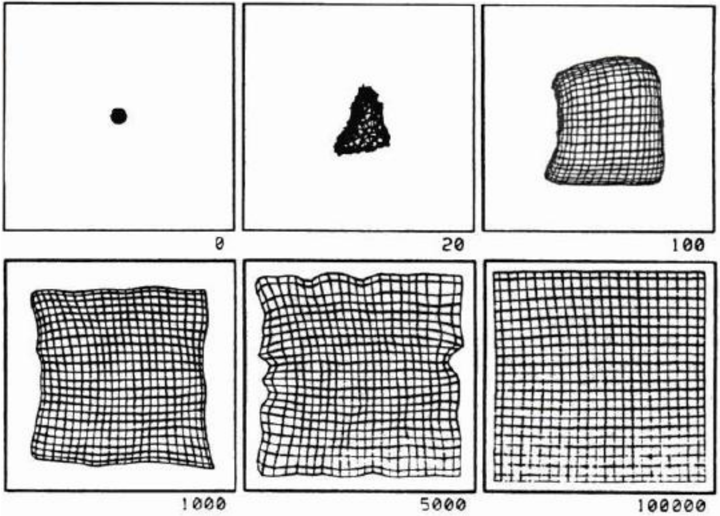
When a pattern $\overrightarrow{\phi_i}$ is presented, each node (represented by outer neurons) in the target space computes its activation $\overrightarrow{\phi}_i^t \overrightarrow{w}$.

Most activated node y^* and weights to its neighbours are updated according to a learning rate $\rho(t)$

$$w_{ki}(t+1) = w_{ki}(t) + \rho(t)\Lambda(|y - y^*|)(\overrightarrow{\phi_i} - w_{ki}(t))$$

$\Lambda(\cdot)$ defines a non-increasing neighbourhood function and $|y - y^*|$ describes the distance of nodes in the neighbourhood

SOM – Self organisation



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SOM – Definition

Condensed definition of SOM from Cottrell et al.[2]

Self organising maps

- Let $I = \{\overrightarrow{\eta_1}, \ldots, \overrightarrow{\eta_{|S|}}\}$ be a set of km-dimensional vectors that are associated with nodes in a lattice.
- Neighbourhood structure provided by symmetrical neighbourhood function $d : I \times I \to \mathbb{R}$ which depends on the distance between two nodes $\overrightarrow{\eta_i}$ and $\overrightarrow{\eta_j} \in I$.
- State of the map at time t given by

$$\eta(t) = \left(\overrightarrow{\eta_1(t)}, \overrightarrow{\eta_2(t)}, \ldots, \overrightarrow{\eta_{|S|}(t)}\right),$$

[2] M. Cottrell, J.C. Fort and G. Pages, *Theoretical aspects of the SOM algorithm*, Neurocomputing, pp. 119-138, vol 21, 1998.

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SOM – Definition

Self organising map algorithm

The SOM algorithm is recursively defined by

$$\begin{aligned} i_c\left(\overrightarrow{v(t+1)}, \overrightarrow{\eta(t)}\right) &= argmin\left\{\left\|\overrightarrow{v(t+1)} - \overrightarrow{\eta_i(t)}\right\|, \overrightarrow{\eta_i(t)} \in \eta(t)\right\}, \\ \overrightarrow{\eta_i(t+1)} &= \overrightarrow{\eta_i(t)} - \varepsilon_t d\left[i_c\left(\overrightarrow{v(t+1)}, \overrightarrow{\eta(t)}\right), \overrightarrow{\eta_i}\right] \\ &\cdot \left(\overrightarrow{\eta_i(t)} - \overrightarrow{v(t+1)}\right), \forall \overrightarrow{\eta_i} \in I. \end{aligned}$$

In this formula, $i_c\left(\overrightarrow{v(t+1)}, \overrightarrow{\eta(t)}\right)$ corresponds to the node in the network that is closest to the input vector.

Parameter ε_t controls the adaptability.

SOM – Operational principle

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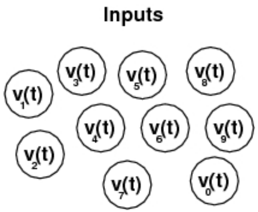

Input values $v_i(t)$ are to be mapped onto the target space

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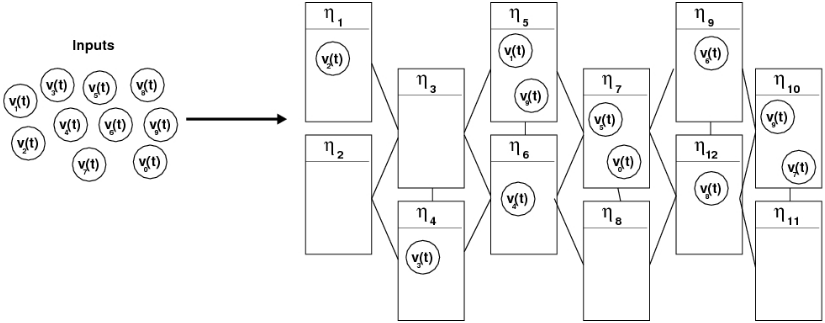
SOM – Operational principle



Node with the lowest distance is associated with the input value:

$$i_c\left(\overrightarrow{v(t+1)}, \overrightarrow{\eta(t)}\right) = argmin\left\{\left\|\overrightarrow{v(t+1)} - \overrightarrow{\eta_i(t)}\right\|, \overrightarrow{\eta_i(t)} \in \eta(t)\right\}$$

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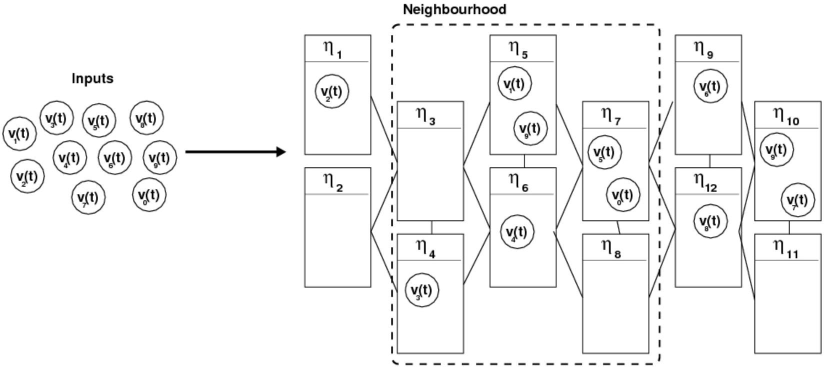
SOM – Operational principle



Nodes in the neighbourhood of the associated node are moved closer to the input value

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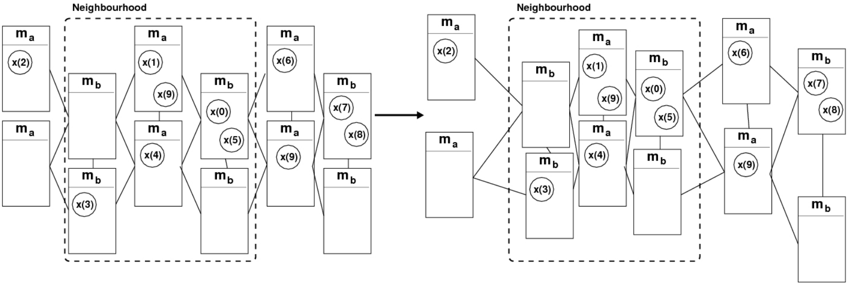
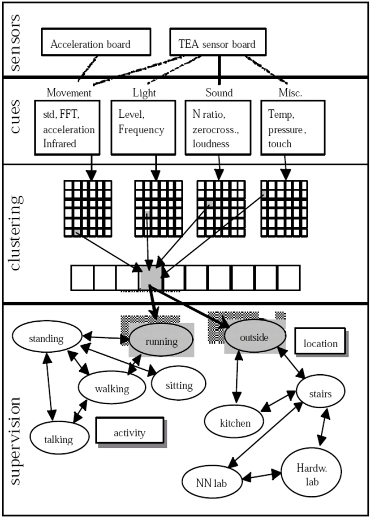
SOM – Operational principle



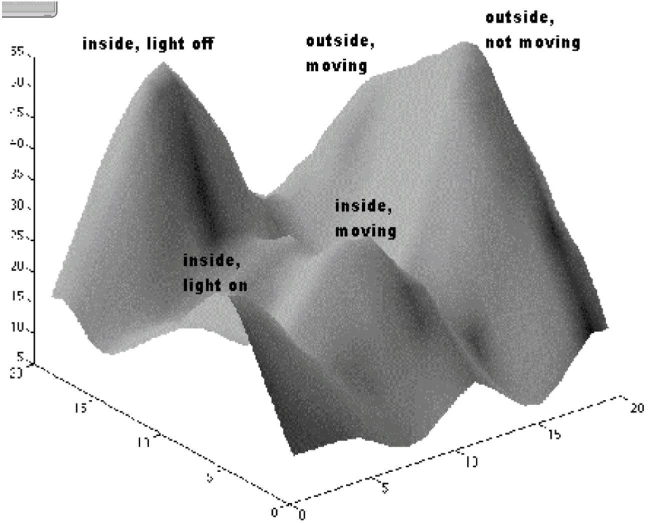
Nodes in the neighbourhood of the associated node are moved to the input value

$$\overrightarrow{\eta_i(t+1)} = \overrightarrow{\eta_i(t)} - \varepsilon_t d \left[i_c \left(\overrightarrow{v(t+1)}, \overrightarrow{\eta(t)} \right), \overrightarrow{\eta_i} \right] \cdot \left(\overrightarrow{\eta_i(t)} - \overrightarrow{v(t+1)} \right), \forall \overrightarrow{\eta_i} \in I.$$

SOM – Example application: TEA



SOM – Example application: TEA



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SOM – Remarks

SOM algorithm always converges[3]

Normalisation of input vectors might improve numerical accuracy

Not guaranteed that self-optimisation will always occur
(Dependent on choice of parameters)

Difficult to set parameters of the model since SOM is not optimising any well-defined function[4]

If neighbourhood is chosen to be too small, the map will not be ordered globally

[3] Y. Cheng, *Neural Computation*, 9(8), 1997.

[4] E. Erwin, K. Obermayer, K. Schulten: Self-organising maps: Ordering, convergence properties and energy functions. Biological Cybernetics, 67, 47-55, 1992

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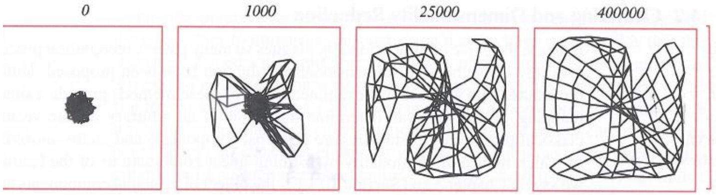
Problems of SOMs



Map created as target space might have several orientations

One part of the map might follow one orientation, while other parts are following other orientations

Questions?

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Literature

- C.M. Bishop: Pattern recognition and machine learning, Springer, 2007.
- R.O. Duda, P.E. Hart, D.G. Stork: Pattern Classification, Wiley, 2001.