### **Selected Topics of Pervasive Computing**

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## Overview and Structure

- 30.10.2013 Organisational
- 30.10.3013 Introduction
- 06.11.2013 Classification methods (Basic recognition, Bayesian, Non-parametric)
- 13.11.2013 Classification methods (Linear discriminant, Neural networks)
- 20.11.2013 -
- 27.11.2013 -
- 04.12.2013 -
- 11.12.2013 Classification methods (Sequential, Stochastic)
- 18.12.2013 Activity Recognition (Basics, Applications, Algorithms, Metrics)
- 08.01.2014 Security from noisy data (Basics, Entity, F. Commitment, F. Extractors)
- 15.01.2014 Security from noisy data (Error correcting codes, PUFs, Applications)
- 22.01.2014 Context prediction (Algorithms, Applications)
- 29.01.2014 Networked Objects (Sensors and sensor networks, body area networks)
- 05.02.2014 Internet of Things (Sensors and Technology, vision and risks)

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### Outline

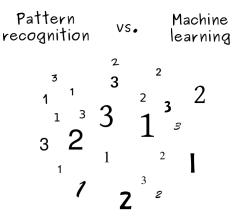
### Introduction

- Recognition of patterns
- Bayesian decision theory
- Non-parametric techniques
- Linear discriminant functions
- Neural networks
- Sequential data
- Stochastic methods
- Conclusion

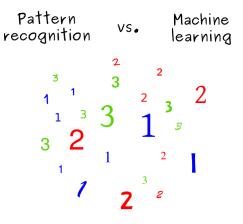
(Introduction)

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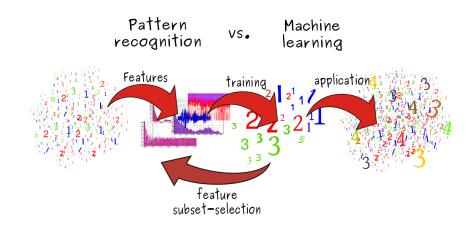


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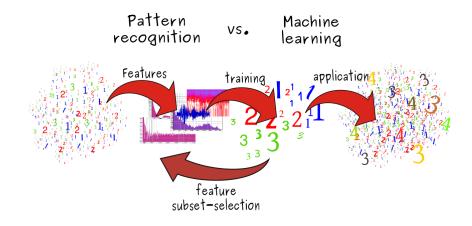
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Introduction



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(Introduction)



- Mapping of features onto classes by using prior knowledge
- What are characteristic features?
- Which approaches are suitable to obtain these features?

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Non-parametric Linear discriminant

Sequential

### Data sampling

- Record sufficient training data
  - Annotated! (Ground-truth)
  - Multiple subjects
  - Various environmental conditions (time of day, weather, ...)



Non-parametric Linear discriminant

# Data sampling

- Record <u>sufficient</u> training data
  - Annotated! (Ground-truth)
  - Multiple subjects
  - Various environmental conditions (time of day, weather, ...)

### Example

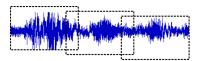
- Electric supply data over 15 years covers 5000 days but only 15 christmas days
- Especially critical events like accidents (e.g. plane, car, earthquake) are scarce



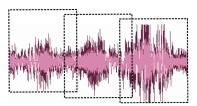
Non-parametric Linear discriminant NN

Sequential

### Feature subset-selection



- Pre-process data
  - Framing
  - Normalisation



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### Feature subset-selection

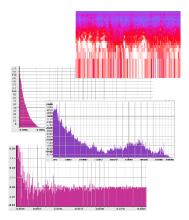
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Domain knowledge?
                 -> better set of
                     ad-hoc features
        Features commensurate?
                 -> normalise
    Pruning of input required?
                 -> if no, create disjunctive
                    features or weithted
                      sums of features
        Independent features?
                -> construct conjunctive features
                     or products of features
             Is the data noisy?
                -> detect outlier examples
Do you know what to do first?
                -> If not, use a linear predictor
```

- Pre-process data
  - Framing
  - Normalisation

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### Feature extraction

- Identify meaningful features
  - remove irrelevant/redundant features

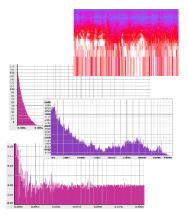


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Feature extraction

- Identify meaningful features
  - remove irrelevant/redundant features
- Features can be contradictory!



### Feature subset-selection

Simple ranking of features with correlation coefficients Example: Pearson Correlation Coefficient

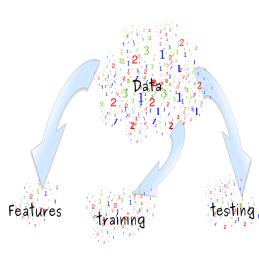
$$\varrho(X,Y) = \frac{\operatorname{Cov}(X,Y)}{\sqrt{\operatorname{Var}(X)\operatorname{Var}(Y)}}$$
(1)

• Identifies linear relation between input variables  $x_i$  and an output y

### Feature subset-selection

# How to do reasonable feature selection

- Utilise dedicated test- and training- data-sets
- Pay attention that a single raw-data sample could not impact features in both these sets
- Don't train the features on the training- or testdata-set



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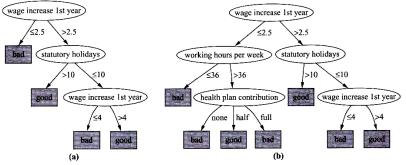
Non-parametric Linear discriminant

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Sequential

### Training of the classifier

### A decision tree classifier



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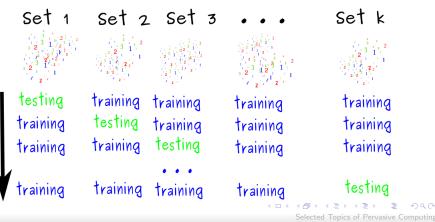
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# Training of the classifier

Evaluation of classification performance

- k-fold cross-validation
  - Standard:  $k{=}10$

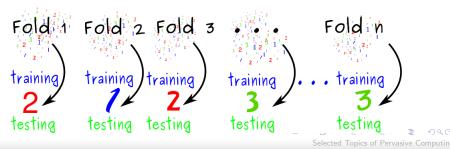


# Training of the classifier

Evaluation of classification performance

Leave-one-out cross-validation

- n-fold cross validation where n is the number of instances in the data-set
- Each instance is left out once and the algorithm is trained on the remaining instances
- Performance of left-out instance (success/failure)



Non-parametric Linear discriminant

Sequential

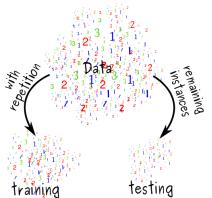
# Training of the classifier

Evaluation of classification performance

0.632 Bootstrap

- Form training set by choosing n instances from the data-set with replacement
- All not picked instances are used for testing
- Probability to pick a specific instance:

$$1 - (1 - \frac{1}{n})'' \approx 1 - e^{-1} \approx 0.632$$



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Non-parametric Linear discriminant

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# Training of the classifier

### Evaluation of classification performance

### Classification accuracy

- Confusion matrices
- Precision
- Recall

	1		Clas	sific	ation			
	Aw	No	P	Sb	S	Sr	St	$\Sigma$
Aw	52		3	6	0	17	22	100
No	-	436	25	7	6	17	9	
To		40	59				1	
Sb	15	22	scorese	32	4	22	5	
SI	12	11	1	6	48	8	14	
Sr	4	15		6	1	67	7	
St	3	18	1	1	24	10	43	
27	92	551	86	65	94	129	83	

1			Cla	ssifica	ation			
	Aw	No	$\mathbf{T}_{0}$	Sb	SI	Sr	St	recall
Aw	.58	.09		.13	.11	.05	.04	
No		.872	.05	.014	.012	.034	.018	
To		.4	.59				.01	
Sb	.15	.22		.32	.04	.22	.05	
SI	.12	.11	.01	.06	.48	.08	.14	
Sr	.04	.15		.06	.01	.67	.07	
St		.18	.01	.01	.24	.1	.43	
prec	.630	.791	.686	.492	.511	.519	.518	

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### Training of the classifier

Evaluation of classification performance

### Information score

Let C be the correct class of an instance and  $\mathcal{P}(C)$ ,  $\mathcal{P}'(C)$  be the prior and posterior probability of a classifier We define:<sup>1</sup>

$$I_{i} = \begin{cases} -\log(\mathcal{P}(\mathcal{C})) + \log(\mathcal{P}'(\mathcal{C})) & \text{if } \mathcal{P}'(\mathcal{C}) \geq \mathcal{P}(\mathcal{C}) \\ -\log(1 - \mathcal{P}(\mathcal{C})) + \log(1 - \mathcal{P}'(\mathcal{C})) & \text{else} \end{cases}$$
(2)

The information score is then

$$\mathsf{IS} = \frac{1}{n} \sum_{i=1}^{n} I_i \tag{3}$$

I. Kononenko and I. Bratko: Information-Based Evaluation Criterion for Classifier's Performance, Machine Learning 6 67-80 1991

Non-parametric Linear discriminant NN

Sequential

### Training of the classifier

Evaluation of classification performance

Brier score

The Brier score is defined as

Brier = 
$$\sum_{i=1}^{n} (t(x_i) - p(x_i))^2$$
 (4)

where

$$t(x_i) = \begin{cases} 1 & \text{if } x_i \text{ is the correct class} \\ 0 & \text{else} \end{cases}$$
(5)

and  $p(x_i)$  is the probability the classifier assigned to the class  $x_i$ .

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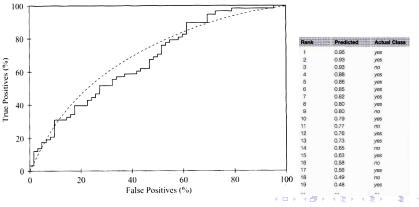
Non-parametric Linear discriminant

Sequential

## Training of the classifier

Evaluation of classification performance

Area under the receiver operated characteristic (ROC) curve (AUC)



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Conclus

# Pattern recognition and classification

### Data mining frameworks

- Orange Data Mining (http://orange.biolab.si/)
- Weka Data Mining (http://www.cs.waikato.ac.nz/ml/weka/)

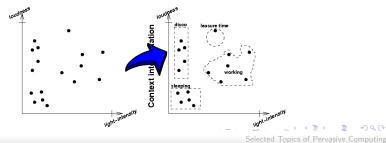




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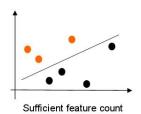
- From features to context.
  - Measure available data on features
  - Context reasoning by appropriate method
    - Syntactical (rule based e.g. RuleML)
    - Bayesian classifier
    - Non-parametric
    - Linear discriminant
    - Neural networks
    - Sequential
    - Stochastic



- Allocation of sensor value by defined function
  - Correlation of various data sources
  - Several methods possible simple approaches
  - Template matching
  - Minimum distance methods
  - 'Integrated' feature extraction
    - Nearest Neighbour
    - Neural Networks
- Problem
  - Measured raw data might not allow to derive all features required
  - Therefore often combination of sensors



Not enough features



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- Methods Syntactical (Rule based)
  - Idea: Description of Situation by formal Symbols and Rules
  - Description of a (agreed on?) world view
  - Example: RuleML
- Comment
  - Pro:
    - Combination of rules and identification of loops and impossible conditions feasible

Contra:

- Very complex with more elaborate situations
- Extension or merge of rule sets typically not possible without contradictions

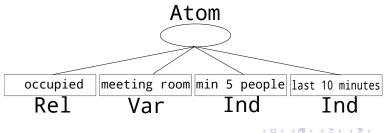
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- Rule Markup Language: Language for publishing and sharing rules
- Hierarchy of rule-sub-languages (XML, RDF, XSLT, OWL)
- Example:
  - A meeting room was occupied by min 5 people for the last 10 minutes.

<atom></atom>			
<rel></rel>	occupied	<td>l&gt;</td>	l>
<var></var>	meeting	room	</td

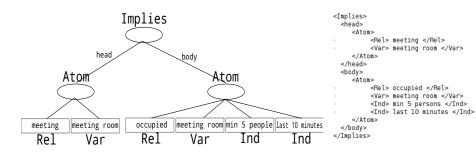
```
<Ind> min 5 persons </Ind>
     <Ind> last 10 minutes </Ind>
</Atom>
```

| room </Var>

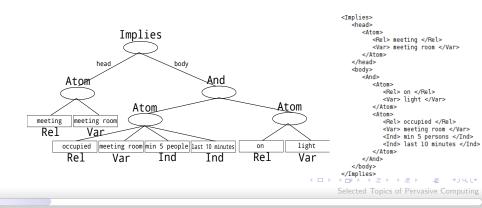


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- Also conditions can be modelled
  - A Meeting is taking place in a meeting room when it was occupied by min 5 people for the last 10 minutes.



- Logical combination of conditions
  - A Meeting is taking place in a meeting room when it was occupied by min 5 people for the last 10 minutes and the light is on.



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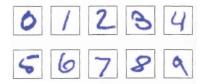
### Recognition of patterns

Patterns can be described by a sufficient number of rules

Samples are inaccurate

Tremendous amount of rules to model all variations of one class

**Therefore:** Consider machine learning approaches



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### Recognition of patterns

Training set  $x_1 \dots x_N$  of a large number of N samples is utilised

Classes  $t_1 \ldots t_N$  of all samples in this set known in advance

Machine learning algorithm computes a function y(x) and generates a new target t'

y(♠) → .1

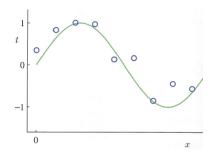
# Polynomial curve fitting

(Recognition)

### Example

A curve shall be approximated by a machine learning approach

Sample points are created for the function  $sin(2\pi x) + N$  where N is a random noise value



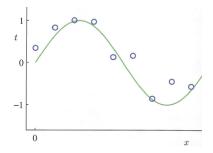
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### Polynomial curve fitting

We will try to fit the data points into a polynomial function:

$$y(x, \overrightarrow{w}) = w_0 + w_1 x + w_2 x^2 + \dots + w_M x^M = \sum_{j=0}^M w_j x^j$$



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We will try to fit the data points into a polynomial function:

$$y(x, \overrightarrow{w}) = w_0 + w_1 x + w_2 x^2 + \dots + w_M x^M = \sum_{j=0}^M w_j x^j$$

This can be obtained by minimising an error function that measures the misfit between  $y(x, \vec{w})$  and the training data set:

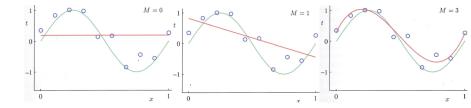
$$E(\overrightarrow{w}) = \frac{1}{2} \sum_{i=1}^{N} \left[ y(x_i, \overrightarrow{w}) - t_i \right]^2$$

 $E(\vec{w})$  is non-negative and zero if and only if all points are covered by the function

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One problem is the right choice of the dimension M

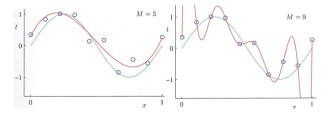
When M is too small, the approximation accuracy might be bad



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However, when M becomes too big, the resulting polynomial will cross all points exactly

When M reaches the count of samples in the training data set, it is always possible to create a polynomial of order M that contains all values in the data set exactly.



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Recognition

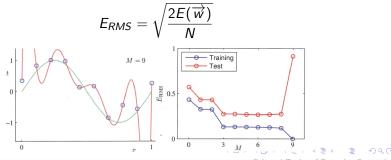
This event is called overfitting

The polynomial now trained too well to the training data

Non-parametric

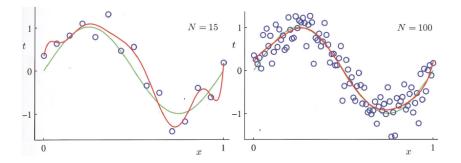
It will therefore perform badly on another sample of test data for the same phenomenon

We visualise it by the Root of the Mean Square (RMS) of  $E(\vec{w})$ 



Linear discriminant

With increasing number of data points, the problem of overfitting becomes less severe for a given value of M



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One solution to cope with overfitting is regularisation

A penalty term is added to the error function

This term discourages the coefficients of  $\vec{w}$  from reaching large values

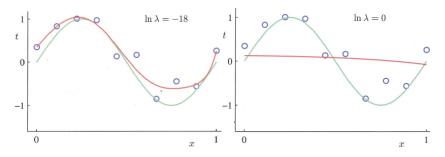
$$\overline{E}(\overrightarrow{w}) = \frac{1}{2} \sum_{i=1}^{N} \left[ y(x_i, \overrightarrow{w}) - t_i \right]^2 + \frac{\lambda}{2} ||\overrightarrow{w}||^2$$

with

$$||\overrightarrow{w}||^2 = \overrightarrow{w}^T \overrightarrow{w} = w_0^2 + w_1^2 + \dots + w_M^2$$

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Depending on the value of  $\lambda$ , overfitting is controlled



$$\overline{E}(\overrightarrow{w}) = rac{1}{2}\sum_{i=1}^{N}\left[y(x_i, \overrightarrow{w}) - t_i
ight]^2 + rac{\lambda}{2}||\overrightarrow{w}||^2$$

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## Outline

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- Recognition of patterns
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## Bayesian decision theory

With probability theory, the probability of events can be estimated by repeatedly generating events and counting their occurrences

When, however, an event only very seldom occurs or is hard to generate, other methods are required

Example:

Probability that the Arctic ice cap will have disappeared by the end of this century

In such cases, we would like to model uncertainty

In fact, it is possible to represent uncertainty by probability



## Conditional probability

#### Conditional probability

The conditional probability of two events  $\chi_1$  and  $\chi_2$  with  $P(\chi_2) > 0$  is denoted by  $P(\chi_1|\chi_2)$  and is calculated by

$$P(\chi_1|\chi_2) = \frac{P(\chi_1 \cap \chi_2)}{P(\chi_2)}$$

 $P(\chi_1|\chi_2)$  describes the probability that event  $\chi_2$  occurs in the presence of event  $\chi_2$ .

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# Bayesian decision theory

With the notion of conditional probability we can express the effect of observed data  $\overrightarrow{t} = t_1, \ldots, t_N$  on a probability distribution of  $\overrightarrow{w}$ :  $P(\overrightarrow{w})$ .

Thomas Bayes described a way to evaluate the uncertainty of  $\overrightarrow{w}$  after observing  $\overrightarrow{t}$ 

$$P(\overrightarrow{w}|\overrightarrow{t}) = \frac{P(\overrightarrow{t}|\overrightarrow{w})P(\overrightarrow{w})}{P(\overrightarrow{t})}$$

 $P(\overrightarrow{t}|\overrightarrow{w})$  expresses how probable a value for  $\overrightarrow{t}$  is given a fixed choice of  $\overrightarrow{w}$ 

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### Bayesian decision theory

A principle difference between Bayesian viewpoint and frequentist viewpoint is that prior assumptions are provided

Example:

Consider a fair coin that scores heads in three consecutive tosses

Classical maximum likelihood estimate will predict head for future tosses with probability  $\ensuremath{\mathbf{1}}$ 

Bayesian approach includes prior assumptions on the probability of events and would result in a less extreme conclusion



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# Bayesian curve fitting

Recognition

In the curve fitting problem, we are given  $\overrightarrow{x}$  and  $\overrightarrow{t}$  together with a new sample  $x_{M+1}$ 

The task is to find a good estimation of the value  $t_{M+1}$ 

This means that we want to evaluate the predictive distribution

$$p(t_{M+1}|x_{M+1}, \overrightarrow{x}, \overrightarrow{t})$$

To account for measurement inaccuracies, typically a probability distribution (e.g. Gauss) is underlying the sample vector  $\vec{x}$ 

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#### (Bayesian)

## Bayesian curve fitting

Recognition

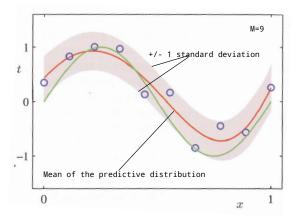
This means that we want to evaluate the predictive distribution

$$p(t_{M+1}|x_{M+1}, \overrightarrow{x}, \overrightarrow{t})$$

After consistent application of the sum and product rules of probability we can rewrite this as

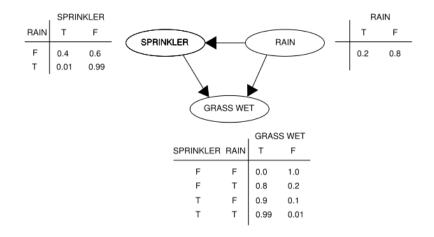
$$p(t_{M+1}|x_{M+1},\overrightarrow{x},\overrightarrow{t}) = \int p(t_{M+1}|x_{M+1},\overrightarrow{w})p(\overrightarrow{w}|\overrightarrow{x},\overrightarrow{t})d\overrightarrow{w}$$

#### Bayesian curve fitting



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### Example



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#### Conclus

## Outline

#### Introduction

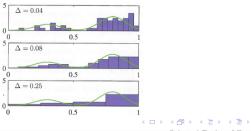
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## Histogram methods

Alternative approach to function estimation: histogram methods

- In general, the probability density of an event is estimated by dividing the range of N values into bins of size  $\Delta_i$
- Then, count the number of observations that fall inside bin  $\Delta_i$
- This is expressed as a normalised probability density

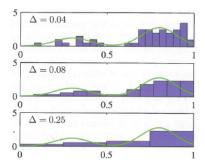
$$p_i = \frac{n_i}{N\Delta_i}$$



#### Histogram methods

Accuracy of the estimation is dependent on the width of the bins

Approach well suited for big data since the data items can be discarded once the histogram is created



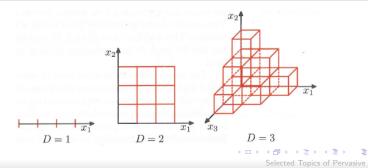
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## Histogram methods

#### Issues:

Due to the edges of the bins, the modelled distribution is characterised by discontinuities not present in the underlying distribution observed

The method does not scale well with increasing dimension (Curse of dimensionality)



(Non-parametric

## Parzen estimator methods

Assume an unknown probability density  $p(\cdot)$ 

We want to estimate the probability density  $p(\vec{x})$  of  $\vec{x}$  in a  $\mathcal{D}$ -dimensional Euclidean space

We consider a small region  $\mathcal{R}$  around  $\overrightarrow{x}$ :

$$P = \int_{\mathcal{R}} p(\overrightarrow{x}) d\overrightarrow{x}$$

We utilise a data set of N observations

Each observation has a probability of P to fall inside  $\mathcal R$ 

With the binomial distribution we can calculate the count K of points falling into  $\mathcal{R}$ :

$$\mathsf{Bin}(K|N,P) = \frac{N!}{K!(N-K)!} P^{K} (1-P)^{N-K}$$

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For large N we can show

 $K \approx NP$ 

With sufficiently small  ${\mathcal R}$  we can also show for the volume V of  ${\mathcal R}$ 

 $P\approx p(\overrightarrow{x})V$ 

Therefore, we can estimate the density as

$$p(\overrightarrow{x}) = \frac{K}{NV}$$

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#### We assume that ${\mathcal R}$ is a small hypercube

In order to count the number K of points that fall inside  $\mathcal{R}$  we define

$$k(\overrightarrow{u}) = \left\{egin{array}{cc} 1, & |u_i| \leq rac{1}{2}, & i=1,\ldots,D, \ 0, & ext{otherwise} \end{array}
ight.$$

This represents a unit cube centred around the origin

This function is an example of a kernel-function or Parzen window

$$k(\overrightarrow{u}) = \left\{ egin{array}{cc} 1, & |u_i| \leq rac{1}{2}, & i = 1, \dots, D, \\ 0, & ext{otherwise} \end{array} 
ight.$$

When the measured data point  $\overrightarrow{x_n}$  lies inside a cube of side *h* centred around  $\overrightarrow{x}$ , we have

$$k\left(\frac{\overrightarrow{x}-\overrightarrow{x_n}}{h}\right)=1$$

The total count K of points that fall inside this cube is

$$K = \sum_{n=1}^{N} k \left( \frac{\overrightarrow{x} - \overrightarrow{x_n}}{h} \right)$$

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The total count K of points that fall inside this cube is

$$K = \sum_{n=1}^{N} k \left( \frac{\overrightarrow{x} - \overrightarrow{x_n}}{h} \right)$$

When we substitute this in the density estimate derived above

$$p(\overrightarrow{x}) = \frac{K}{NV}$$

with volume  $V = h^D$  we obtain the overall density estimate as

$$p(\overrightarrow{x}) = \frac{1}{N} \sum_{n=1}^{N} \frac{1}{h^{D}} \left( \frac{\overrightarrow{x} - \overrightarrow{x_{n}}}{h} \right)$$

$$p(\overrightarrow{x}) = \frac{1}{N} \sum_{n=1}^{N} \frac{1}{h^{D}} \left( \frac{\overrightarrow{x} - \overrightarrow{x_{n}}}{h} \right)$$

Again, this density estimator suffers from artificial discontinuities (Due to the fixed boundaries of the cubes)

Problem can be overcome by choosing a smoother kernel function (A common choice is a Gaussian kernel with a standard deviation  $\sigma$ )

$$p(\overrightarrow{x}) = \frac{1}{N} \sum_{n=1}^{N} \frac{1}{(2\pi\sigma^2)^{\frac{D}{2}}} e^{-\frac{||\overrightarrow{x} - \overrightarrow{x_n}||^2}{2\sigma^2}}$$

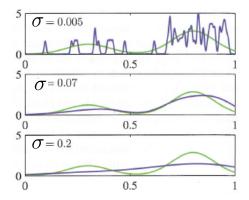
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(Non-parametric

#### Parzen estimator methods

Density estimation for various values of  $\sigma$ 

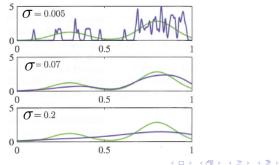


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A problem with Parzen estimator methods is that the parameter governing the kernel width (*h* or  $\sigma$ ) is fixed for all values  $\overrightarrow{x}$ 

#### In regions with

...high density, a wide kernel might lead to over-smoothing ...low density, the same width may lead to noisy estimates



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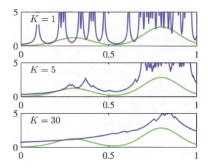
#### NN-methods address this by adapting width to data density

Parzen estimator methods fix V and determine K from the data Nearest neighbour methods fix K and choose V accordingly

Again, we consider a point  $\overrightarrow{x}$  and estimate the density  $p(\overrightarrow{x})$ 

The radius of the sphere is increased until K data points (the nearest neighbours) are covered

The value K then controls the amount of smoothing Again, an optimum value for K exists



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<u>Classification</u>: Apply KNN-density estimation for each class Then, utilise Bayes theorem Assume data set of N points with  $N_k$  points in class  $C_k$ To classify sample  $\overrightarrow{x}$ , draw a sphere containing K points around  $\overrightarrow{x}$ Sphere can contain other points regardless of their class Assume sphere has volume V and contains  $K_k$  points from  $C_k$