

# Machine Learning and Pervasive Computing

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10.12.2014

## Overview and Structure

- 22.10.2014 Organisation
- 22.10.3014 Introduction (Def.: Machine learning, Supervised/Unsupervised, Examples)
- 29.10.2014 Machine Learning Basics (Toolchain, Features, Metrics, Rule-based)
- 05.11.2014** A simple Supervised learning algorithm
- 12.11.2014 Excursion: Avoiding local optima with random search
- 19.11.2014 –
- 26.11.2014** Bayesian learner
- 03.12.2014 –
- 10.12.2014 Decision tree learner
- 17.12.2014** k-nearest neighbour
- 07.01.2015 Support Vector Machines
- 14.01.2015** Artificial Neural networks and Self Organizing Maps
- 21.01.2015 Hidden Markov models and Conditional random fields
- 28.01.2015** High dimensional data, Unsupervised learning
- 04.02.2015 Anomaly detection, Online learning, Recom. systems

# Outline

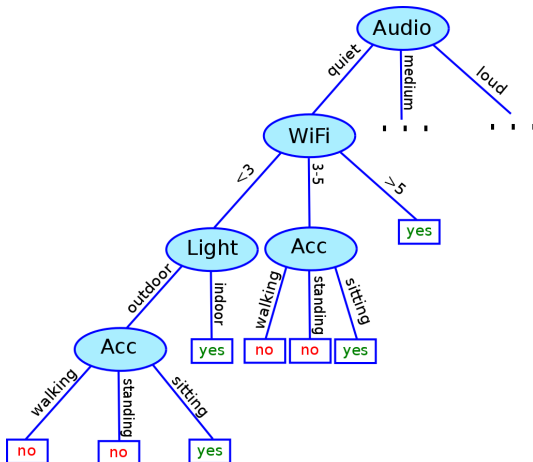
Decision Tree

C4.5

Confidence on a prediction

## Decision tree

A decision tree is a tree that divides the examples from a dataset according to the features and classes observed for them



# Decision tree

How to generate such decision tree?

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- First** select a feature to split on and place it at the root node.
- Then** repeat this procedure for all child nodes

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## How to determine the feature to split on?

# Decision tree

	WiFi		Accelerometer			Audio			Light			At work	
	yes	no	yes	no		yes	no		yes	no	yes	no	
<3 APs	3	7	walking	4	8	quiet	8	5	outdoor	4	7	16	14
[3, 5]	5	5	standing	1	4	medium	6	3	indoor	12	7		
>5 APs	8	2	sitting	11	2	loud	2	6					



# Decision tree

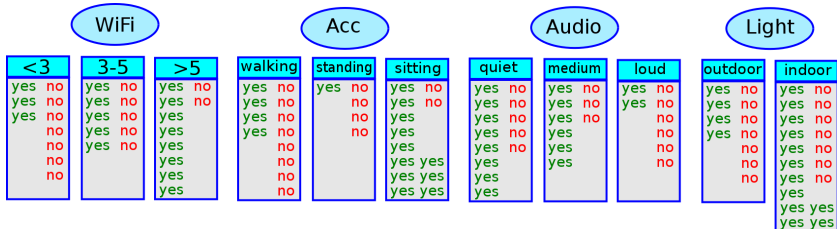
WiFi			Accelerometer			Audio			Light			At work	
	yes	no		yes	no		yes	no		yes	no	yes	no
<3 APs	3	7	walking	4	8	quiet	8	5	outdoor	4	7	16	14
[3, 5]	5	5	standing	1	4	medium	6	3	indoor	12	7		
>5 APs	8	2	sitting	11	2	loud	2	6					

WiFi

<3		3-5		>5	
yes	no	yes	no	yes	no
yes	no	yes	no	yes	no
yes	no	yes	no	yes	
no	no	yes	no	yes	
no	no	yes	no	yes	
no	no			yes	
no	no			yes	

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<3 APs	3	7	walking	4	8	quiet	8	5	outdoor	4	7	16	14
[3, 5]	5	5	standing	1	4	medium	6	3	indoor	12	7		
>5 APs	8	2	sitting	11	2	loud	2	6					



Which one is the best choice?

## Decision tree

WiFi			Acc			Audio			Light	
<3	3-5	>5	walking	standing	sitting	quiet	medium	loud	outdoor	indoor
yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no
yes no	yes no	yes no	yes no	no no	yes no	yes no	yes no	yes no	yes no	yes no
yes no	yes no	yes	yes no	no	yes	yes no	yes no	no	yes no	yes no
no	yes no	yes	yes	no	yes	yes no	yes	no	yes	yes no
no	yes no	yes	no	no	yes	yes no	yes	no	no	yes no
no		yes	no		yes yes	yes	yes	no	no	yes no
no		yes	no		yes yes	yes	yes	no	no	yes no
no		yes	no		yes yes	yes	yes	no	no	yes yes
		yes	no		yes yes	yes	yes	no	no	yes yes
		yes	no		yes yes	yes	yes	no	no	yes yes

We are interested in the gain in information when a particular choice is taken

## Decision tree

WiFi			Acc			Audio			Light	
<3	3-5	>5	walking	standing	sitting	quiet	medium	loud	outdoor	indoor
yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no
yes no	yes no	yes no	yes no	no no	yes no	yes no	yes no	yes no	yes no	yes no
yes no	yes no	yes	yes no	no	yes	yes no	yes no	no	yes no	yes no
no	yes no	yes	yes	no	yes	yes no	yes	no	yes	yes no
no	yes no	yes	no	no	yes	yes no	yes	no	no	yes no
no		yes	no		yes yes	yes	yes	no	no	yes no
no		yes	no		yes yes	yes	yes	no	no	yes no
no		yes	no		yes yes	yes	yes	no	no	yes yes
		yes	no		yes yes	yes	yes	no	no	yes yes

We are interested in the gain in information when a particular choice is taken

The decision tree should then decide for the split that promises maximum information gain.



## Decision tree

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<3	3-5	>5	walking	standing	sitting	quiet	medium	loud	outdoor	indoor
yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no	yes no
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yes no	yes no	yes	yes no	no no	yes	yes no	yes no	no	yes no	yes no
no	yes no	yes	yes no	no	yes	yes no	yes	no	yes no	yes no
no	yes	yes	no	no	yes	yes no	yes	no	no	yes no
no	yes	yes	no	no	yes yes	yes	yes	no	no	yes no
no		yes	no	no	yes yes	yes	yes	no	no	yes
		yes	no	no	yes yes	yes	yes	no	no	yes
		yes	no	no	yes yes	yes	yes	no	no	yes
		yes	no	no	yes yes	yes	yes	no	no	yes

$$\mathcal{E}(p_1, p_2, \dots, p_n) = -p_1 \log_2 p_1 - p_2 \log_2 p_2 \dots - p_n \log_2 p_n$$

WiFi information value:

$$\mathcal{E}\left(\frac{3}{10}, \frac{7}{10}\right) \frac{10}{30} + \mathcal{E}\left(\frac{5}{10}, \frac{5}{10}\right) \frac{10}{30} + \mathcal{E}\left(\frac{8}{10}, \frac{2}{10}\right) \frac{10}{30} =$$

## Decision tree

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no no	yes no	yes no	no no	no no	yes yes	yes no	yes	no no	no no	yes no
no no	yes no	yes no	no no	no no	yes yes	yes		no	no	yes no
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WiFi information value:

$$\begin{aligned} \mathcal{E}\left(\frac{3}{10}, \frac{7}{10}\right) \frac{10}{30} + \mathcal{E}\left(\frac{5}{10}, \frac{5}{10}\right) \frac{10}{30} + \mathcal{E}\left(\frac{8}{10}, \frac{2}{10}\right) \frac{10}{30} = & \left(-\frac{3}{10} \log_2 \frac{3}{10} - \frac{7}{10} \log_2 \frac{7}{10}\right) \cdot \frac{10}{30} \\ & + \left(-\frac{5}{10} \log_2 \frac{5}{10} - \frac{5}{10} \log_2 \frac{5}{10}\right) \cdot \frac{10}{30} \\ & + \left(-\frac{8}{10} \log_2 \frac{8}{10} - \frac{2}{10} \log_2 \frac{2}{10}\right) \cdot \frac{10}{30} \end{aligned}$$







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yes no	yes no	yes	yes no		yes	yes no	yes no	yes	yes no	yes no
no no	yes no	yes	yes no		yes	yes no	yes	no	yes no	yes no
no no	yes no	yes	no no		yes yes	yes	yes	no	no no	yes no
no no	yes no	yes	no no		yes yes	yes	yes	no	no no	yes no
no		yes	no		yes yes	yes			no	yes
			no			yes			yes	yes
			no						yes	yes

Information value:

Information gain:

WiFi:  $\approx$  0.868Acc:  $\approx$  0.756Audio:  $\approx$  0.884Light:  $\approx$  0.948

(Initial information value (working [yes/no]): 0.997

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no no	yes no	yes no	yes no	no	yes	yes no	yes no	no	yes no	yes no
no no	yes no	yes	no		yes	yes no	yes	no	no	yes no
no no	yes no	yes	no		yes yes	yes	yes	no	no	yes no
no		yes	no		yes yes	yes		no	no	yes no
		yes	no		yes yes	yes			no	yes
		yes	no		yes yes	yes			no	yes yes
		yes	no		yes yes	yes			no	yes yes

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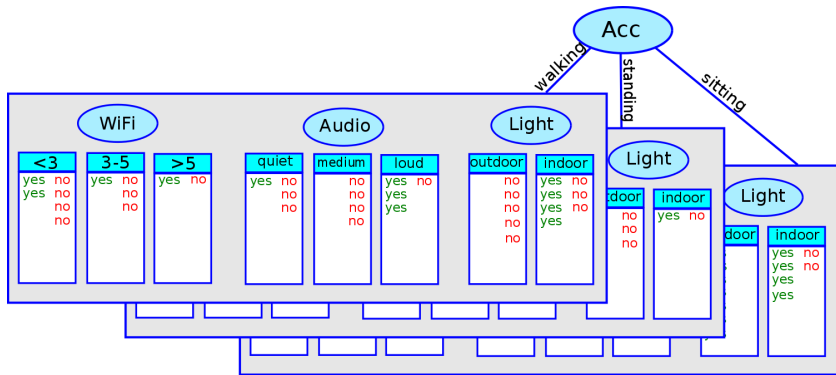
Information gain:

WiFi:  $\approx$  0.129Acc:  $\approx$  0.241Audio:  $\approx$  0.113Light:  $\approx$  0.049

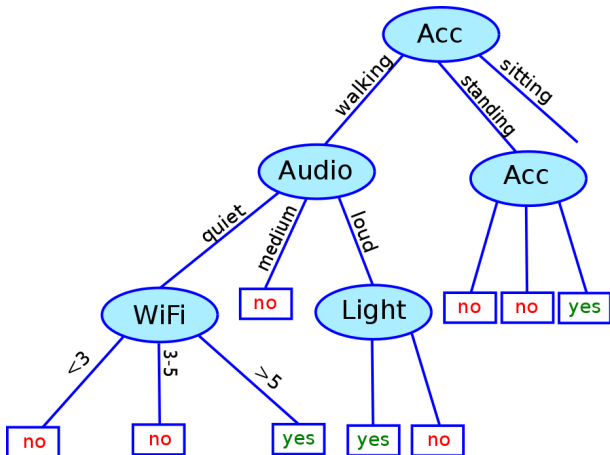
(Initial information value (working [yes/no]): 0.997



## Decision tree



# Decision tree



# Outline

Decision Tree

C4.5

Confidence on a prediction

## Decision tree – C4.5

Improved decision tree implementation: C4.5

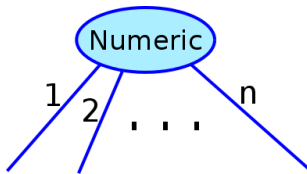
- Dealing with numeric values
- Missing values
- Noisy data



## C4.5 – Dealing with numeric values

### Nominal feature values

For nominal features, the decision tree splits on every possible value. Therefore, the information content of this feature is 0 after such branch has been conducted



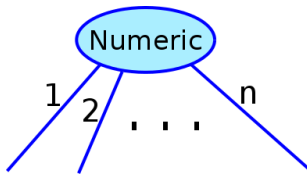
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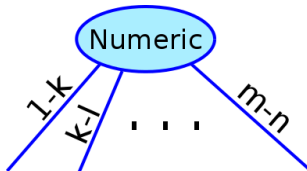
### Numeric feature values

For numeric feature values, splitting on each possible value would lead to a very wide tree of small depth.



## C4.5 – Dealing with numeric values

For numeric values, the tree is split into several intervals.



## C4.5 – Missing values

### Missing values in a data set

Missing values are a common/prominent event in real-world data sets

- participants in a survey refuse to answer
- malfunctioning sensor nodes
- Biology: plants or animals might die before all variables have measured
- ...

Most machine learning schemes make the implicit assumption that there is no significance in the fact that a certain value is missing.

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<sup>1</sup>Witten et al., Data Mining, Morgan Kaufmann, 2011

## C4.5 – Missing values

The absence of data might already hold valuable information!

### Example

People analyzing medical databases have noticed that cases may, in some circumstances, be diagnosable simply from the tests that a doctor decides to make – regardless of the outcome of the tests<sup>1</sup>

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## C4.5 – Missing values

### Possible solution

Considering whether the sets of samples with values have significant difference in their final outcome when compared to the sets of samples that feature missing values



## C4.5 – Noisy data

Fully expanded decision trees often contain unnecessary structure that should be simplified before deployment

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### Pruning

- Prepruning** Trying to decide through the tree-building process when to stop developing subtrees
- Might speed up tree creation phase
  - Difficult to spot dependencies between features at this stage (features might be meaningful together but not on their own)

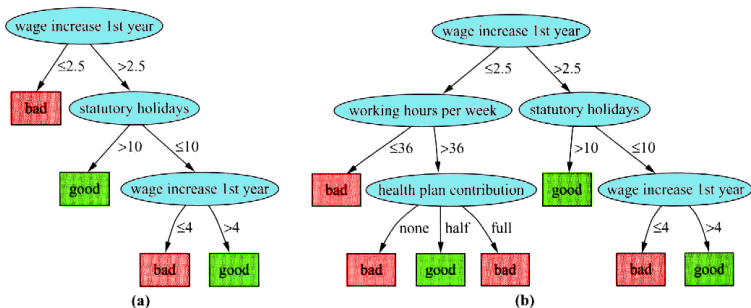
**Postpruning** Simplification of the decision tree after the tree has been created

## C4.5 – Noisy data

### Postpruning – subtree replacement

Select some subtrees and replace them with single leaves

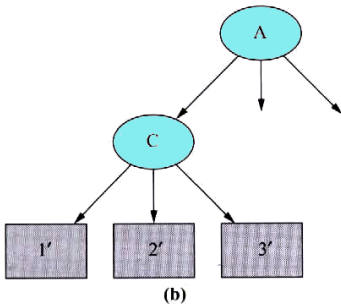
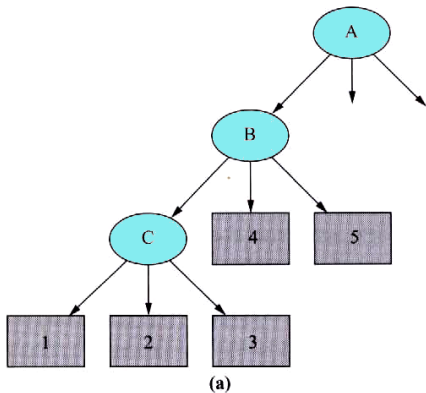
- Will cause accuracy on the training set to decrease
- May increase accuracy on independently chosen test set (reduction of noise)



## C4.5 – Noisy data

### Postpruning – subtree raising

Complete subtree is raised one level and samples at the nodes of the subtree have to be recalculated



## C4.5 – Estimating error rates

When should we raise or replace subtrees?

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Estimation of error rates at internal nodes and leaf nodes.

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- Will lead to a certain number of errors  $E$
- ... out of the total number of instances  $N$



## C4.5 – Estimating error rates

When should we raise or replace subtrees?

### Estimating error rates

Estimation of error rates at internal nodes and leaf nodes.

**Assumption:** Label of node is chosen as the majority vote from all its leaves

- Will lead to a certain number of errors  $E$
- ... out of the total number of instances  $N$
- Assume:
  - 1 True probability of error at that node is  $q$
  - 2  $N$  instances are generated by Bernoulli process with parameter  $q$  and errors  $E$

## C4.5 – Estimating error rates

### Estimating error rates – Calculating the success probability

Given a confidence  $c$  (C4.5 uses 25%), we find a confidence limit  $z$  (for  $c = 25\% \rightarrow z = 0.69$ ) such that

$$\mathcal{P} \left[ \frac{q' - q}{\sqrt{\frac{q(1-q)}{N}}} > z \right] = c$$

(with the observed error rate  $q' = \frac{E}{N}$ )

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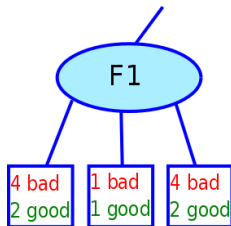
- This leads to an upper confidence limit for  $q$  which we can use to estimate a pessimistic error rate  $e$

$$e = \frac{q' + \frac{z^2}{2N} + z \sqrt{\frac{q'}{N} - \frac{q'^2}{N} + \frac{z^2}{4N^2}}}{1 + \frac{z^2}{N}}$$

## C4.5 – Estimating error rates

### Example

Lower left leaf ( $E = 2, N = 6$ ) Utilising the formula for  $e$ , we obtain  $q' = 0.33$  and  $e = 0.47$



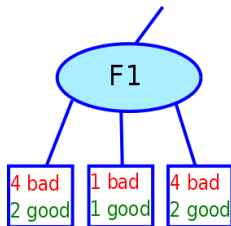
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Center leaf ( $E = 1, N = 2$ )  $e = 0.72$



## C4.5 – Estimating error rates

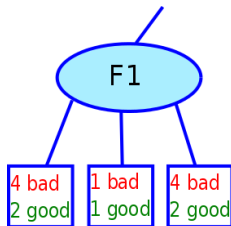
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Center leaf ( $E = 1, N = 2$ )  $e = 0.72$

Right leaf ( $E = 2, N = 6$ )  $e = 0.47$



## C4.5 – Estimating error rates

### Example

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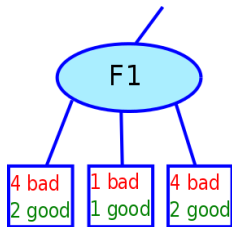
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Right leaf ( $E = 2, N = 6$ )  $e = 0.47$

Combine Error estimates Utilising ratio 6:2:6 this leads to a combined error estimate of

$$\frac{0.47 \cdot 6}{14} + \frac{0.72 \cdot 2}{14} + \frac{0.47 \cdot 6}{14} \approx 0.51$$



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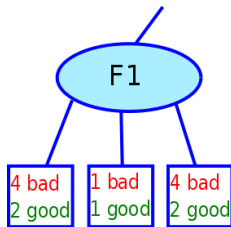
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Error estimate for parent node  $q' = \frac{4}{14} \rightarrow e = 0.46$   
 $0.46 < 0.51 \Rightarrow$  prune children away





## C4.5 – Further heuristics employed

Postpruning – Confidence value  $c = 25\%$

Postpruning – Split Threshold Candidate splits on a numeric feature are only considered when at least  $\min(10\%, 25)$  of all training samples are cut off by the split

Prepruning with information gain Given  $S$  candidate splits on a certain numeric attribute,  $\log_2 \frac{S}{N}$  is subtracted from the information gain

- in order to prevent overfitting
- When information gain is negative, tree-construction will stop

## C4.5 – Remarks

### Postpruning

- Postpruning in C4.5 is very fast and therefore popular
- However, the statistical assumptions are shaky
  - use of upper confidence limit
  - assumption of normal distribution for error rate calculation
  - use of statistics from the training set
- Often, the algorithm does not prune enough and a better performance can be achieved with a more compact decision tree

# Outline

Decision Tree

C4.5

Confidence on a prediction

## Confidence on a prediction

Assume we measure the error of a classifier on a test set and obtain a numerical error rate of  $q$  (a success rate of  $p = (1 - q)$ ).

What can we say about the true success rate?

- It will be close to  $p$ ,
- but how close? (within 5% or 10% ?)

This depends on the size of the test set

Naturally, we are more confident on the success probability  $p$  when it were based on a large number of values.

## Confidence on a prediction

In statistics, a succession of independent events that either succeed or fail is called a **Bernoulli process**

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**Assume** that out of  $N$  events,  $S$  are successful.

**Then** we have an observed success rate of  $p' = \frac{S}{N}$

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**Then** we have an observed success rate of  $p' = \frac{S}{N}$

What can we say about the true success rate  $p$ ?

### Confidence Interval

The answer is expressed as a confidence interval:  
 $p$  lies within a certain specified interval with a certain specified confidence

## Confidence on a prediction

For a specific Bernoulli trial with success rate  $p$  we have

mean  $p$

variance  $p(1 - p)$

For large  $N$ , the distribution of this random variable approaches the normal distribution

## Confidence on a prediction

The probability that a random variable  $X$ , with zero mean, lies within a certain confidence range of width  $2z$  is

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Confidence limits for the normal distribution are e.g.

$\mathcal{P}[\chi \geq z]$	0.001	0.005	0.01	0.5	0.1	0.2	0.4
$z$	3.09	2.58	2.33	1.65	1.28	0.84	0.25

## Confidence on a prediction

The probability that a random variable  $X$ , with zero mean, lies within a certain confidence range of width  $2z$  is

$$\mathcal{P}[-z \leq \chi \leq z] = c$$

Confidence limits for the normal distribution are e.g.

$\mathcal{P}[\chi \geq z]$	0.001	0.005	0.01	0.5	0.1	0.2	0.4
$z$	3.09	2.58	2.33	1.65	1.28	0.84	0.25

Standard assumption in such tables on the random variable:

mean 0  
variance 1

## Confidence on a prediction

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The  $z$  figures are measured in standard deviations from the mean:

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E.g. the figure for  $\mathcal{P}[\chi \geq z] = 0.05$  implies that there is a 5% chance that  $\chi$  lies more than 1.65 standard deviations above the mean.

Since the distribution is symmetric, the chance that  $X$  lies more than 1.65 standard deviations from the mean is 10%:

$$\mathcal{P}[-1.65 \leq \chi \leq 1.65] = 0.9$$



## Confidence on a prediction

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This leads to

$$\mathcal{P} \left[ -z < \frac{p' - p}{\sqrt{\frac{p(1-p)}{N}}} < z \right] = c$$

## Confidence on a prediction

To find confidence limits, given a particular confidence figure  $c$ :

- consult a table with confidence limits for the normal distribution for the corresponding value  $z$
- Note: since Success probabilities are displayed, we have to subtract our value  $c$  from 1 and divide by two:

$$\frac{1 - c}{2}$$

- Then, write the inequality above as an equality and invert it to find an expression for  $p$
- Finally, solving a quadratic equation will produce the respective value for  $p$

## Confidence on a prediction

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$$p = \frac{\left( p' + \frac{z^2}{2N} V \pm z \sqrt{\frac{p'}{N} - \frac{p'^2}{N} + \frac{z^2}{4N^2}} \right)}{1 + \frac{z^2}{N}}$$

The resulting two values are the upper and lower confidence boundaries

# Confidence on a prediction

## Example

$$p' = 0.75; N = 1000, c = 0.8 (z = 1.28) \rightarrow [0.732, 0.767]$$

$$p' = 0.75; N = 100, c = 0.8 (z = 1.28) \rightarrow [0.691, 0.801]$$

Note that the assumptions taken are only valid for large  $N$

# Outline

Decision Tree

C4.5

Confidence on a prediction

# Questions?

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## Literature

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- R.O. Duda, P.E. Hart, D.G. Stork: Pattern Classification, Wiley, 2001.

