Machine Learning and Pervasive Computing

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Overview and Structure

13.04.2015	Organisation
13.04.2015	Introduction
20.04.2015	Rule-based learning
27.04.2015	Decision Trees
04.05.2015	A simple Supervised learning algorithm
11.05.2015	–
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25.05.2015	–
01.06.2015	High dimensional data
08.06.2015	Artificial Neural Networks
15.06.2015	k-Nearest Neighbour methods
22.06.2015	Probabilistic graphical models
29.06.2015	Topic models
06.07.2015	Unsupervised learning
13.07.2015	Anomaly detection, Online learning, Recom. systems

Outline

Anomaly detection

Problem statement

Anomaly detection

Anomaly detection is the task to identify anomalous behaviour with respect to behaviour trained on a data set of prior samples

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Examples: Fraud detection Identify users with unusual behaviour (e.g. non-human profiles in a web-application)

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Examples:
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- Manufacturing: Identify faulty engines (e.g. too much vibration, too loud, ...)

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- **Computers in a datacenter** Identify machines that do not work properly (e.g. processing load over network traffic)

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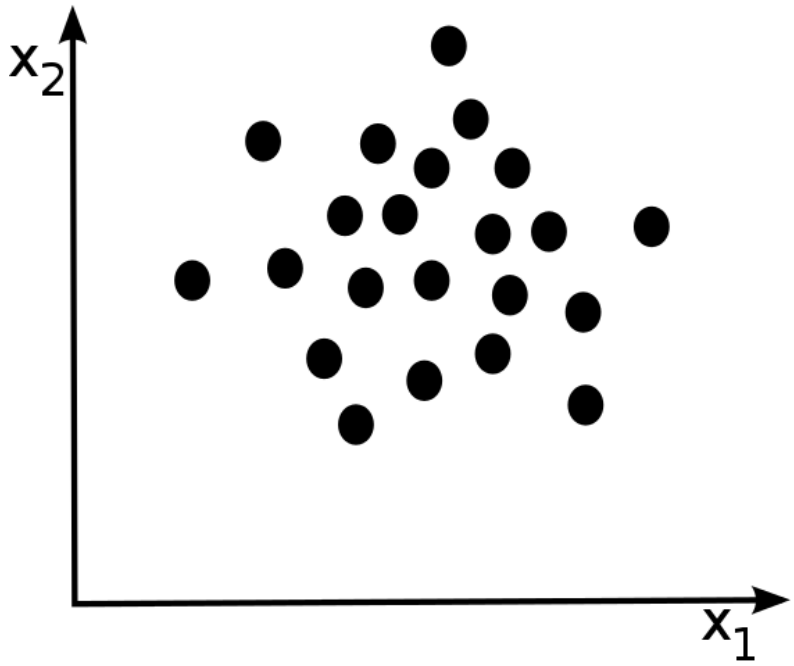
- **Manufacturing** Identify faulty engines (e.g. too much vibration, too loud, ...)
- **Computers in a datacenter** Identify machines that do not work properly (e.g. processing load over network traffic)
- **Network traffic** Identify unusual traffic patterns (e.g. traffic amount or origin)

Anomaly detection

Problem statement



Anomaly detection

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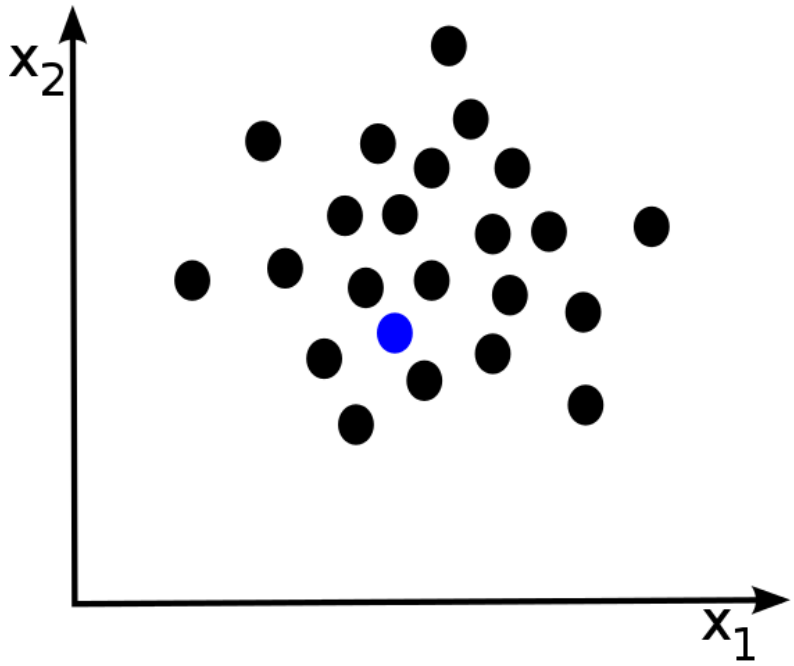

Anomaly detection

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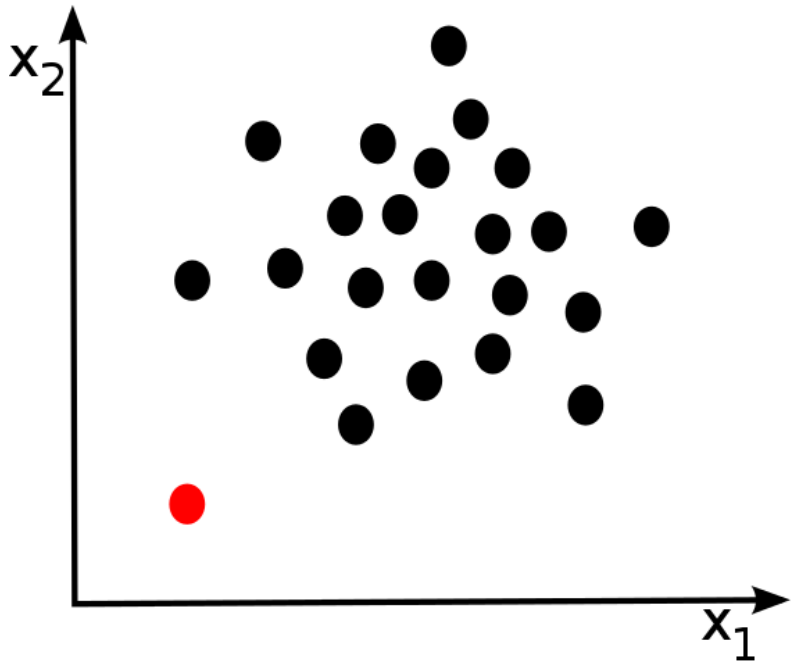

Anomaly detection

Anomaly detection algorithm

1. Choose features that are indicative of anomalous examples

Anomaly detection

Anomaly detection algorithm

1. Choose features that are indicative of anomalous examples
2. Conditioning on a Gaussian distribution, fit the parameters

$$\mu_j = \frac{1}{n}\sum_{i=1}^{n} x_j^{(i)} \quad \text{(mean of a Gaussian distribution)}$$

$$\sigma_j^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_j^{(i)} - \mu_j\right)^2 \quad \text{(standard deviation of a Gaussian distribution)}$$

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3. For a new sample x, compute

$$\mathcal{P}[x] = \prod_{j=1}^{m} P[x_j; \mu_j, \sigma_j^2] = \prod_{j=1}^{m} \underbrace{\frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j-\mu_j)^2}{2\sigma_j^2}}}_{\text{Gaussian distribution}}$$

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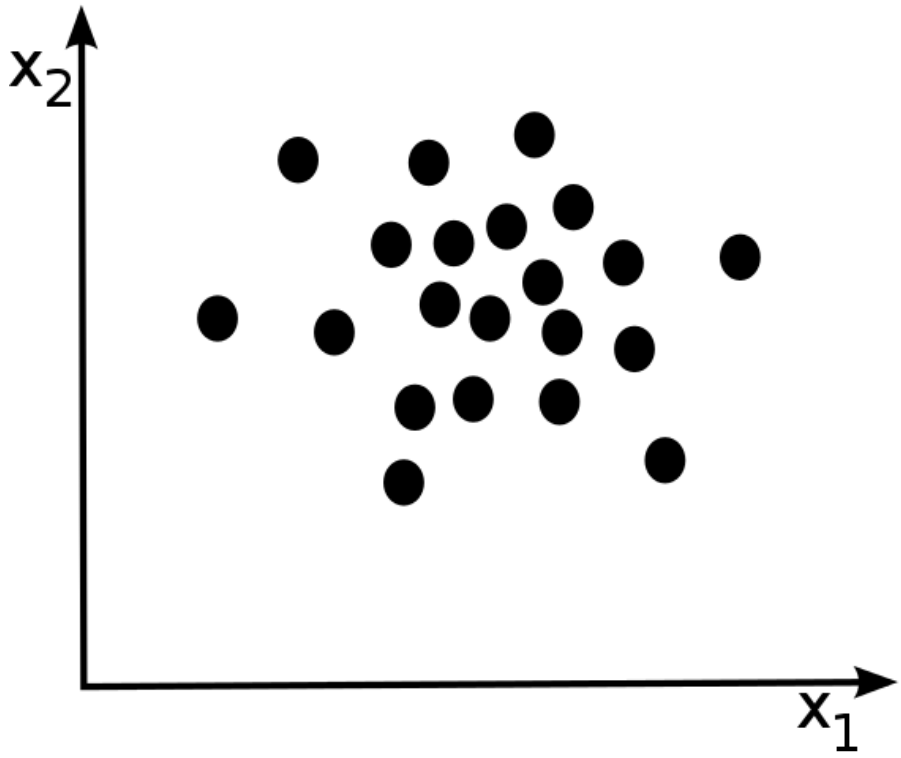
4. anomaly if $\mathcal{P}[x] < \varepsilon$

Anomaly detection

Example



Anomaly detection

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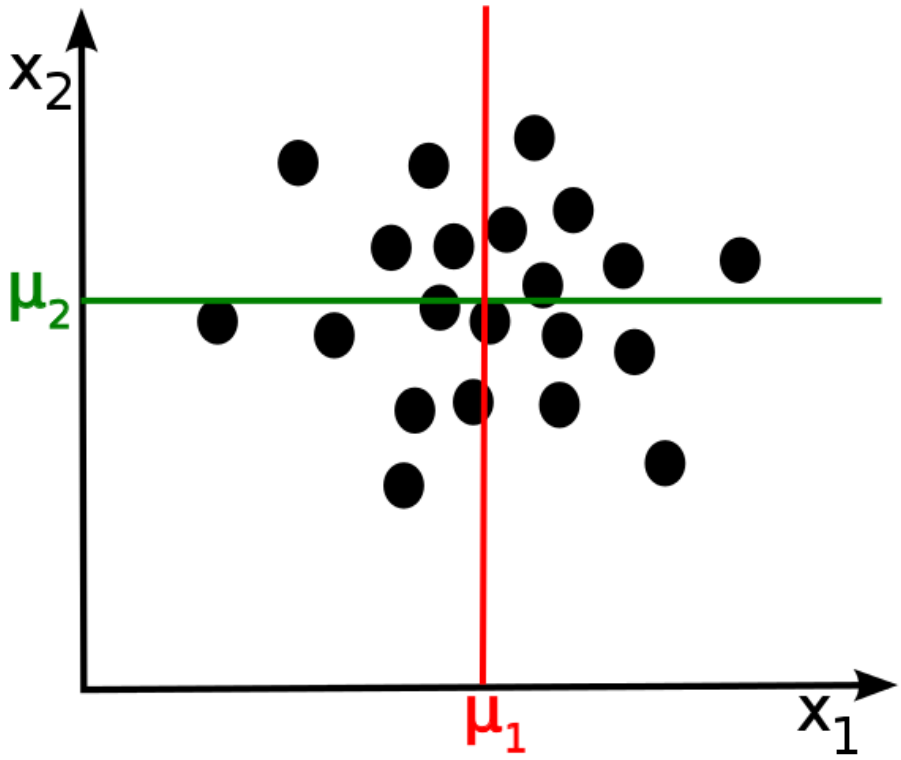

Anomaly detection

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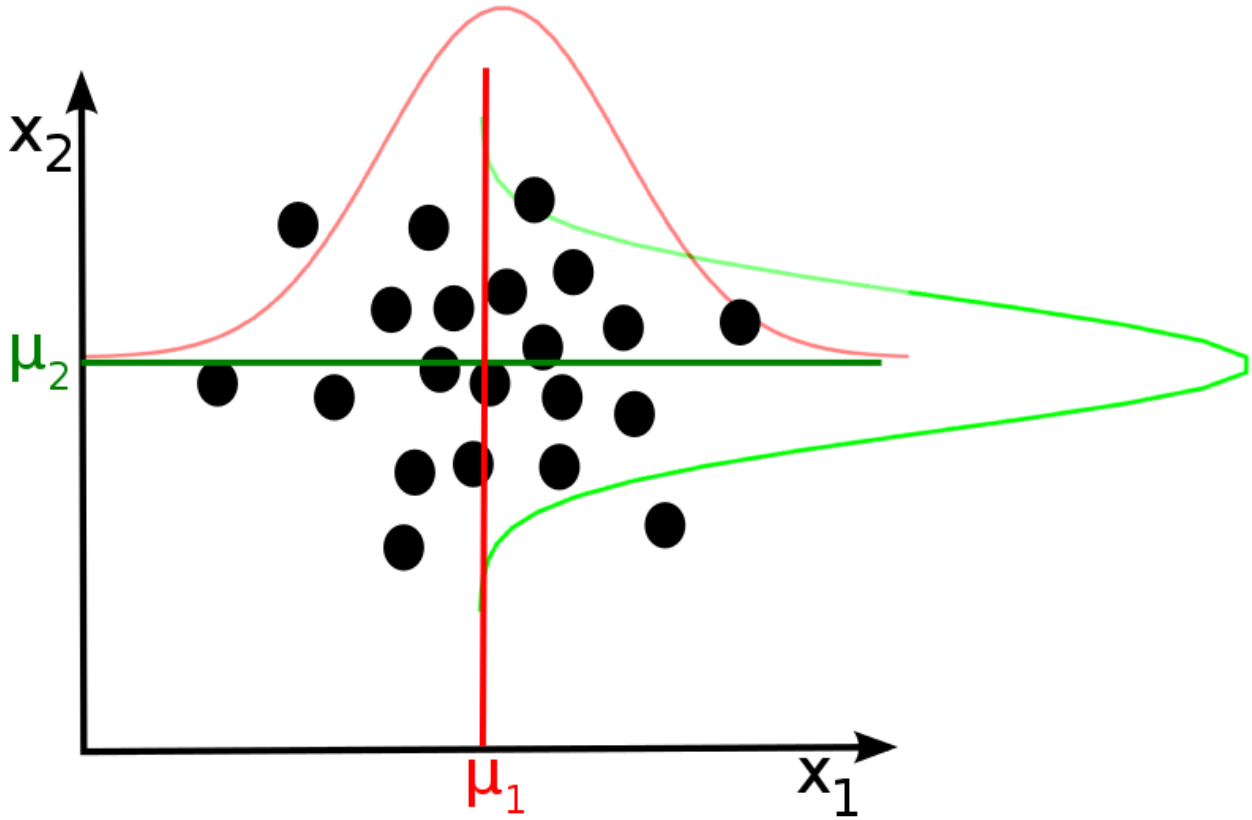
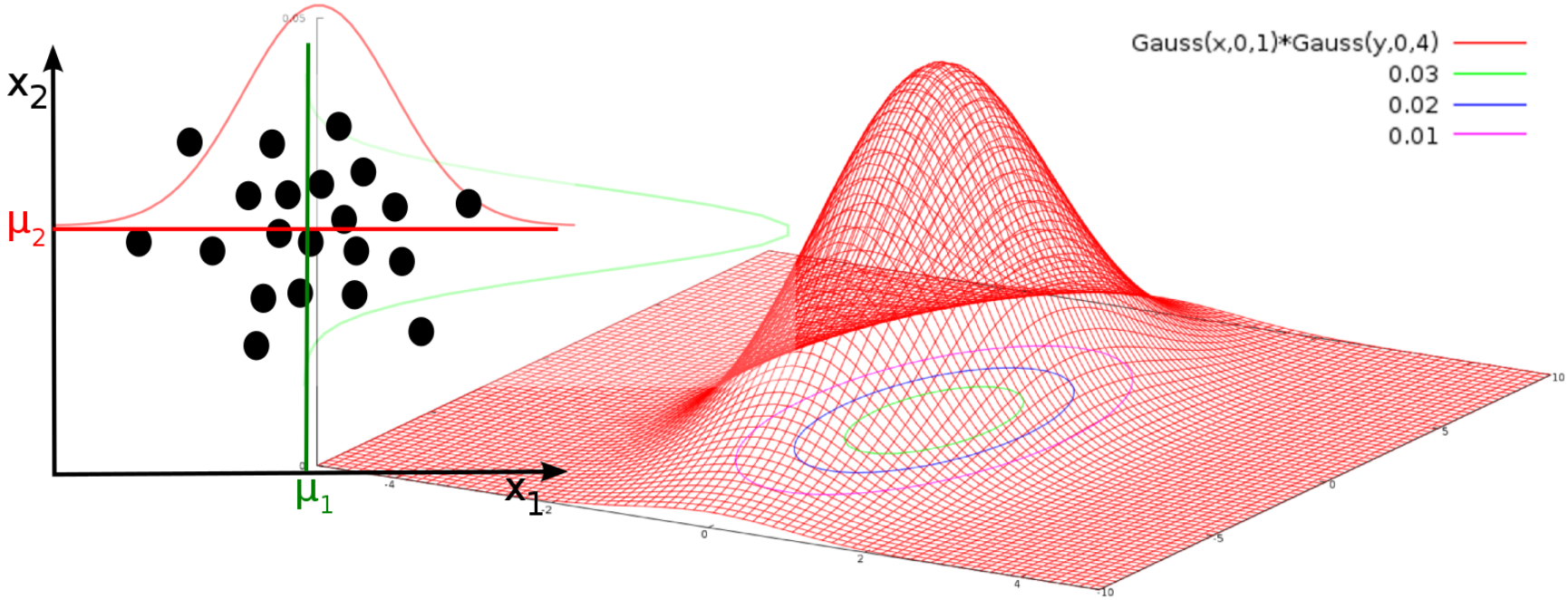


Anomaly detection

Example



Anomaly detection

Problem statement

Choice of good values for ε

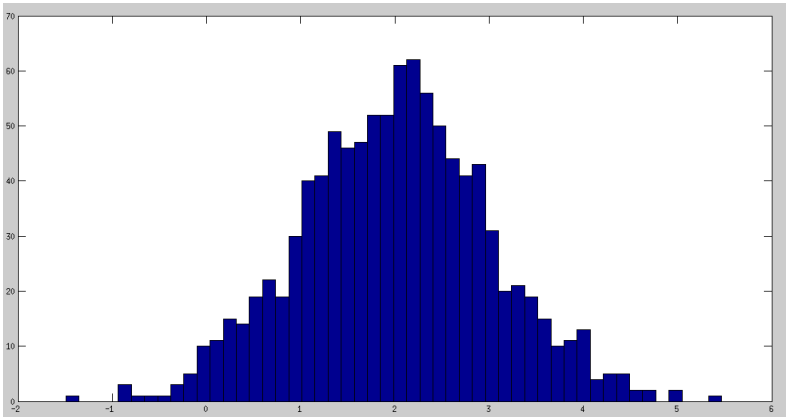
Using crossvalidation and testing sets, calculate

- Precision/Recall
- F_1-score
- ...

Anomaly detection

Non-Gaussian features

In anomaly detection, we have so far assumed Gaussian distributed features.

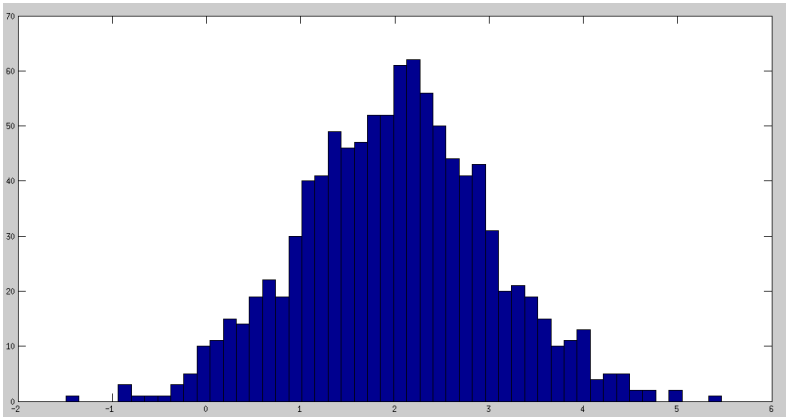


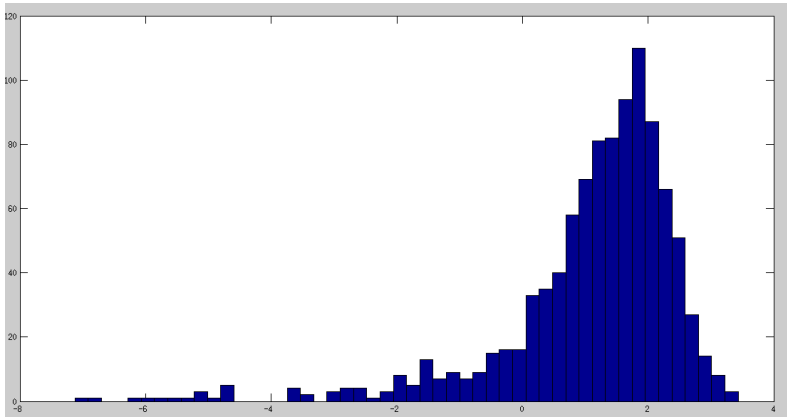
Anomaly detection

Non-Gaussian features

In anomaly detection, we have so far assumed Gaussian distributed features.

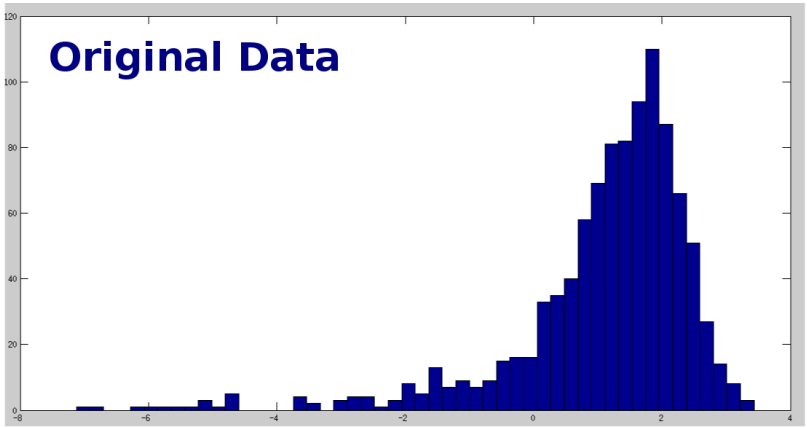
→ What if the feature distribution is not Gaussian ?

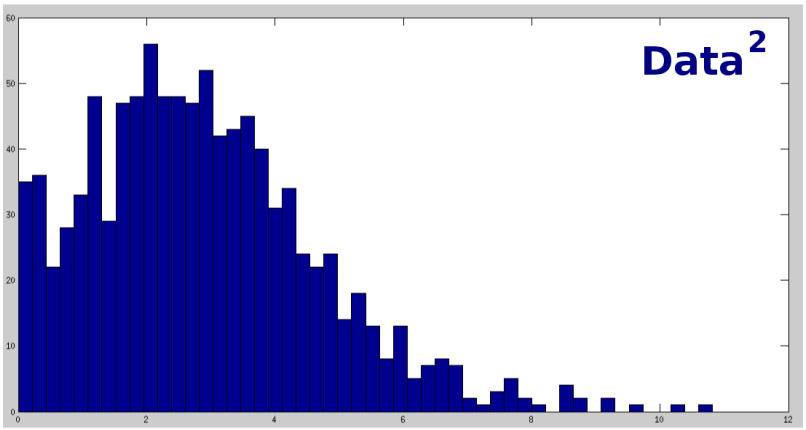


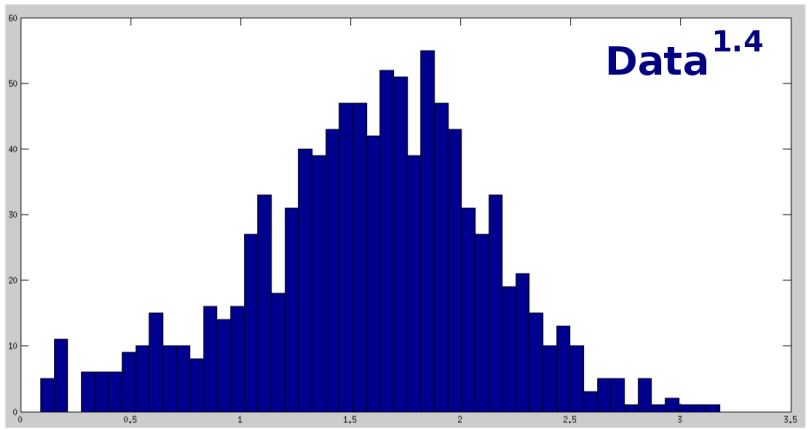


Anomaly detection

Generate new features with a more Gaussian-like distribution







Anomaly detection

Non-Gaussian features

Possible operations on features

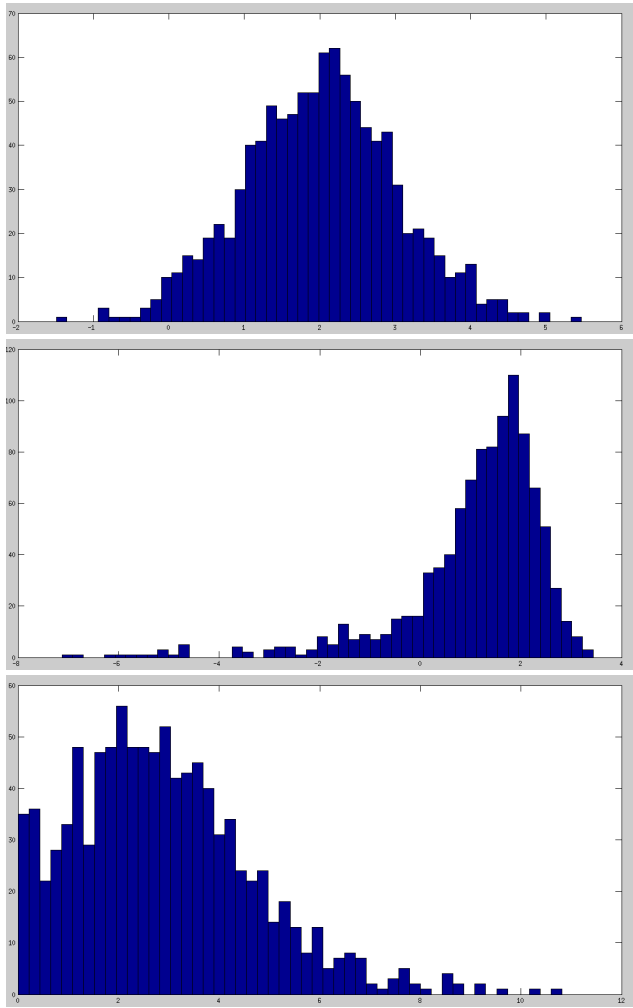
$$x_{\text{new}} = \log(x)$$

$$x_{\text{new}} = \sqrt{x}$$

$$x_{\text{new}} = x^{\frac{1}{3}}$$

$$x_{\text{new}} = \log(x + k)$$

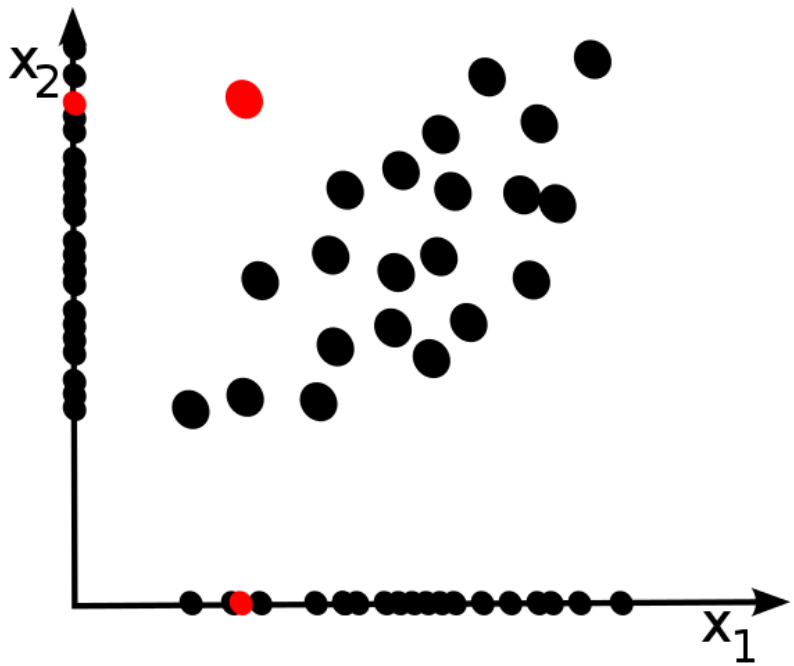
$$\vdots$$



Anomaly detection

Multivariate Gaussian Distribution

- Note that there are cases in which the anomaly looks perfectly normal when considering each dimension separately

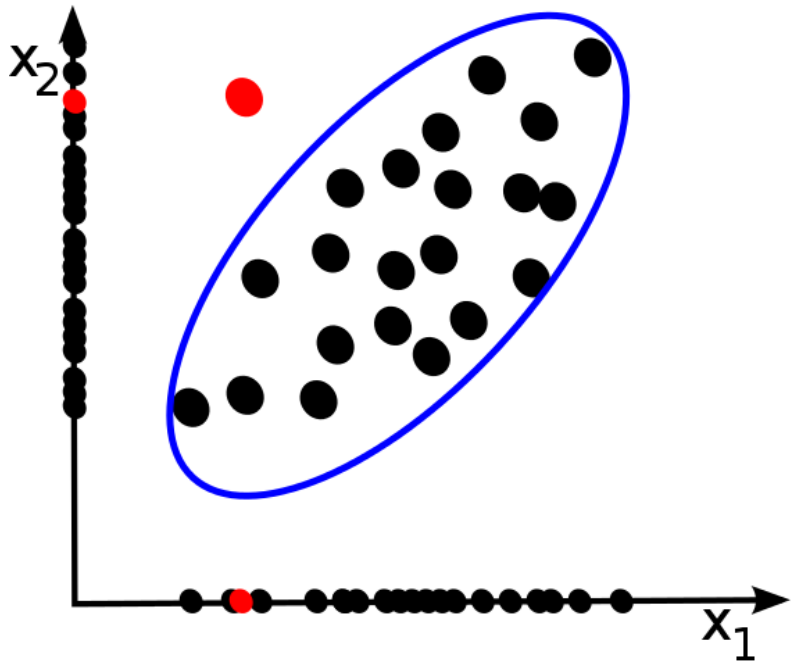


Anomaly detection

Multivariate Gaussian Distribution

- Note that there are cases in which the anomaly looks perfectly normal when considering each dimension separately

→ The consideration of multivariate Gaussian distributions might be suggestive in order to detect such anomalies.



Outline

Elektronik

USB Datenkabel für ... (1) EUR 15,00 Warum empfohlen?
LG E960 E 960 Nexus ... (11) EUR 71,69 Warum empfohlen?
GiXa Technology ... (8) EUR 5,99 Warum empfohlen?
Original Google Nexus ... (55) EUR 88,88 Warum empfohlen?
Werkzeug Reparatur ... (22) EUR 13,90 Warum empfohlen?
Schraubenzieher ... (18) ~~EUR 7,88~~ EUR 7,15 Warum empfohlen?

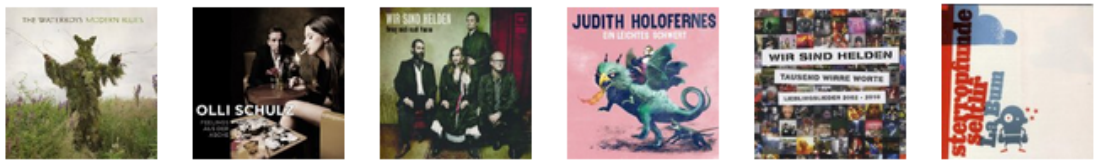
› Alle Empfehlungen in Elektronik anzeigen

Computer & Zubehör

Logitech C270 USB HD ... (1.138) ~~EUR 29,99~~ EUR 22,37 Warum empfohlen?
EDIMAX EW-7811UN ... (611) EUR 7,99 Warum empfohlen?
Präzision Werkzeugset ... (28) EUR 5,39 Warum empfohlen?
Drahtloses Qi ... (135) EUR 18,99 Warum empfohlen?
Anker® [3-Pack] ... (257) EUR 9,99 Warum empfohlen?
Salcar® TV Stick ... (71) ~~EUR 16,76~~ EUR 13,18 Warum empfohlen?

› Alle Empfehlungen in Computer & Zubehör anzeigen

Musik



Neuerscheinung
Modern Blues
› The Waterboys
(5)
EUR 14,99
Warum empfohlen?

Neuerscheinung
Feelings aus der Asche
› Olli Schulz
(54)
EUR 14,99
Warum empfohlen?

Bring mich nach Hause
› Wir Sind Helden
(101)
EUR 3,80
Noch 7 Stück auf Lager.
Warum empfohlen?

Ein Leichtes Schwert
› Judith Holofernes
(51)
EUR 12,39
Noch 11 Stück auf Lager.
Warum empfohlen?

Tausend Wirre Worte ...
› Wir Sind Helden
(18)
EUR 6,99
Noch 18 Stück auf Lager.
Warum empfohlen?

La Bum (Re-Release)
› Sportfreunde Stiller
(39)
EUR 12,12
Noch 1 Stück auf Lager.
Warum empfohlen?

› Alle Empfehlungen in Musik anzeigen

Bücher

Citrus 01
Saburouta

Bedeutung und ...
James W. Heisig

Wir sind Helden: ...
Wir sind Helden

Die Kana lernen und ...
James W. Heisig

Vokabelkarten ...
Hefei Huang

Kanji und Kana: Die ...
Wolfgang Hadamitzky

Recommender systems

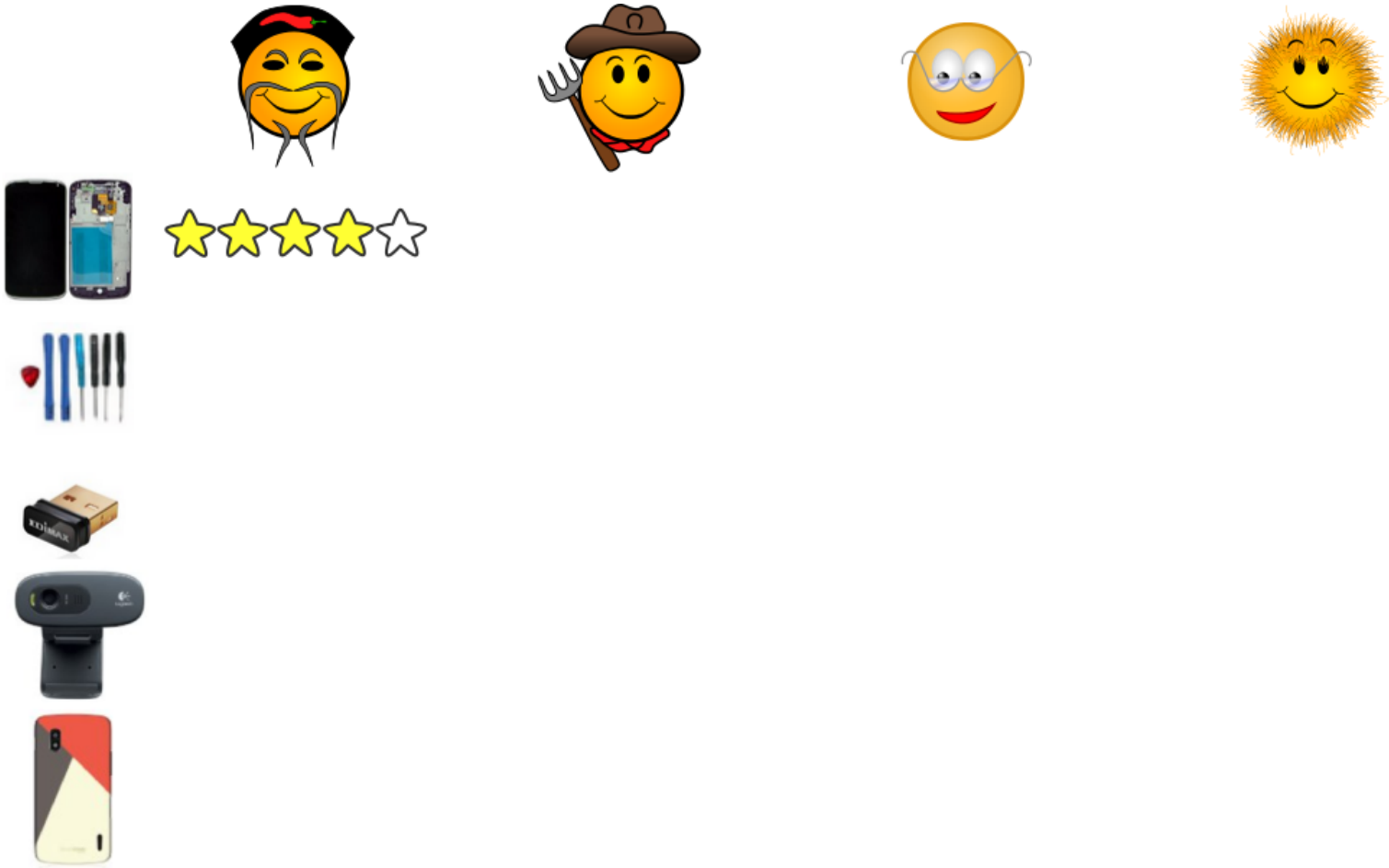




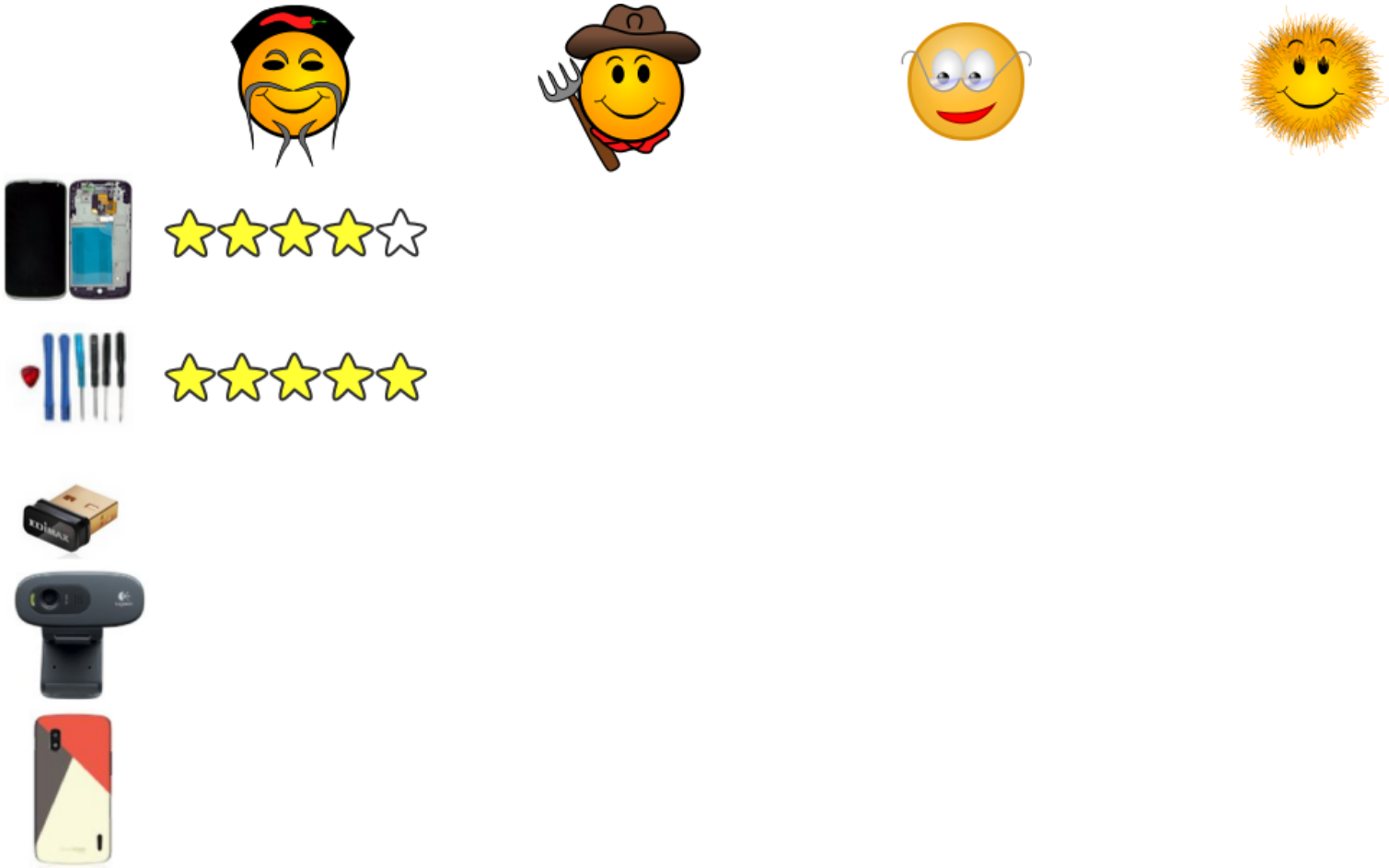




Recommender systems



Recommender systems



Recommender systems

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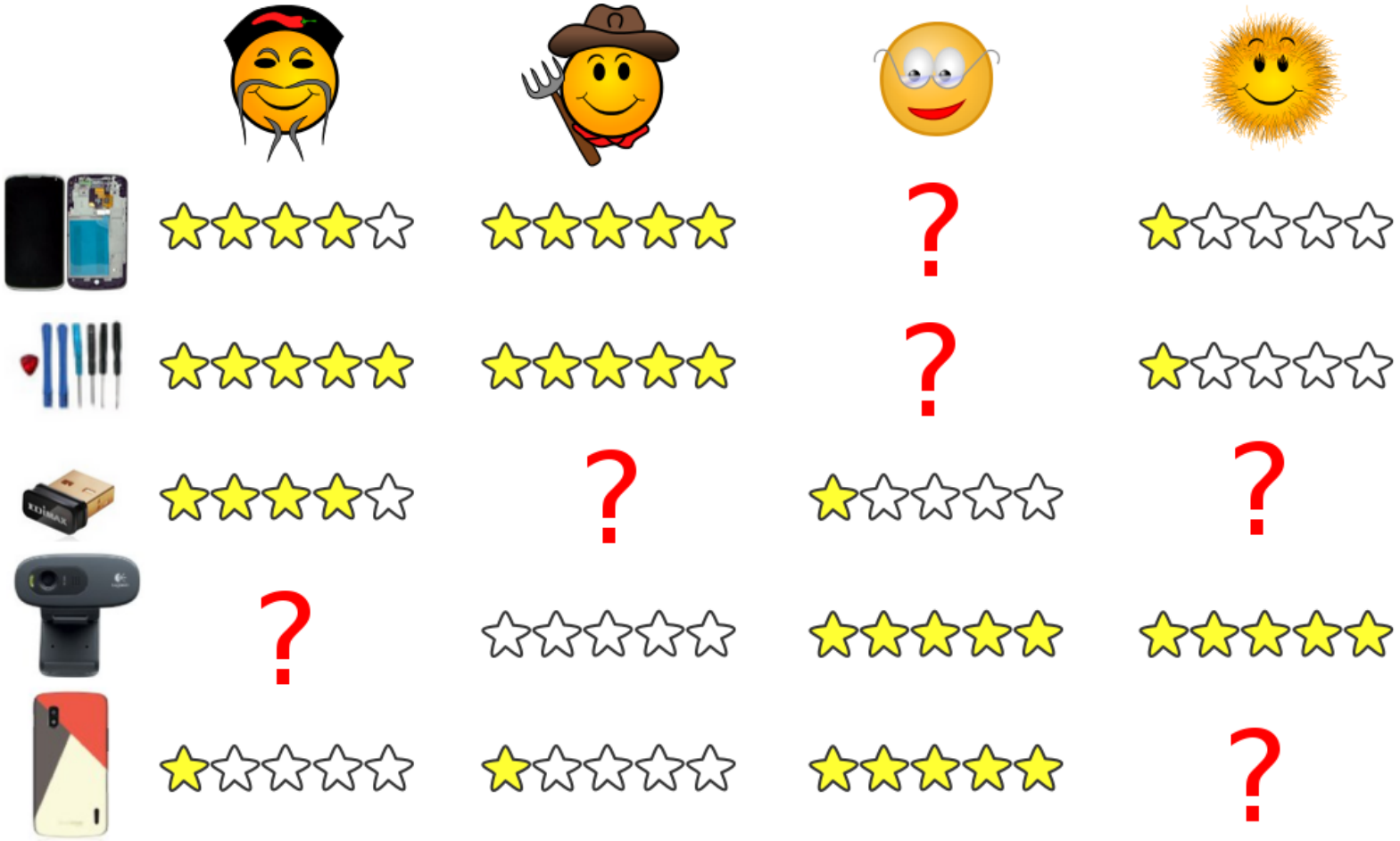
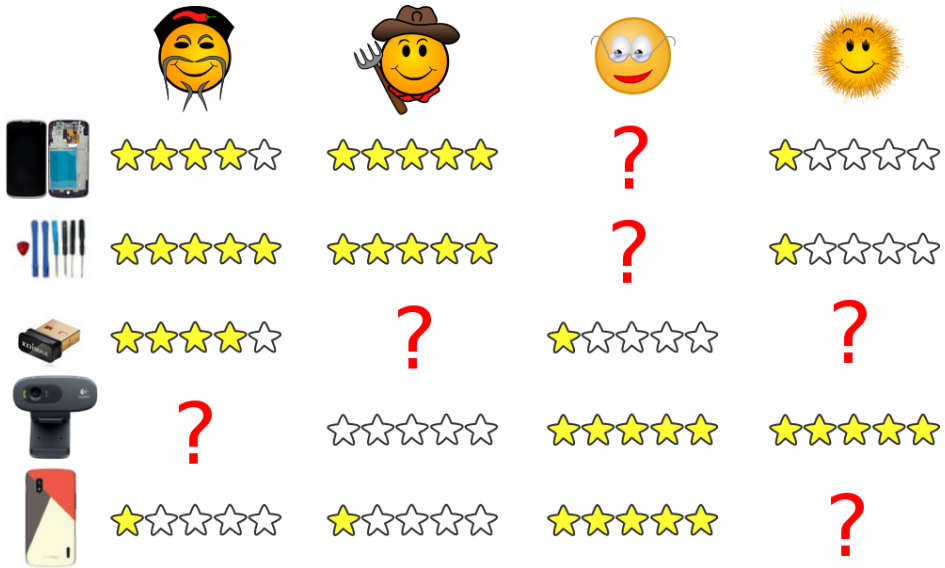
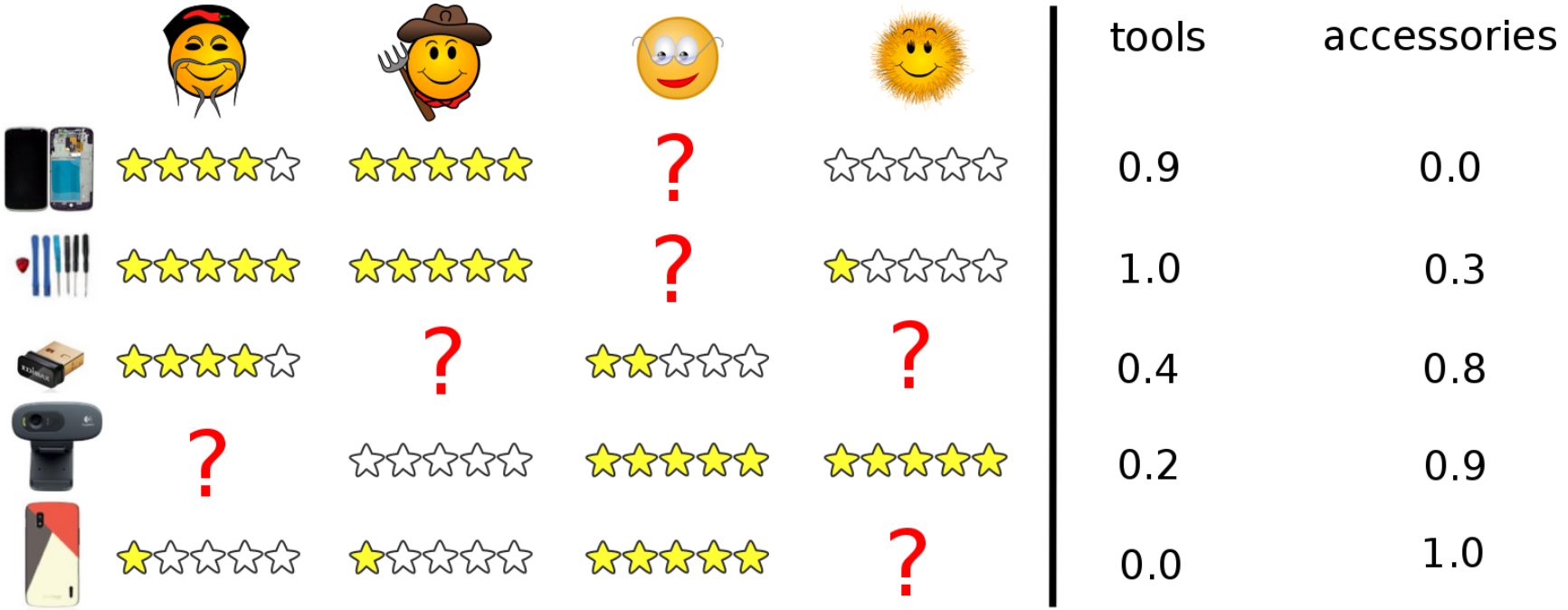


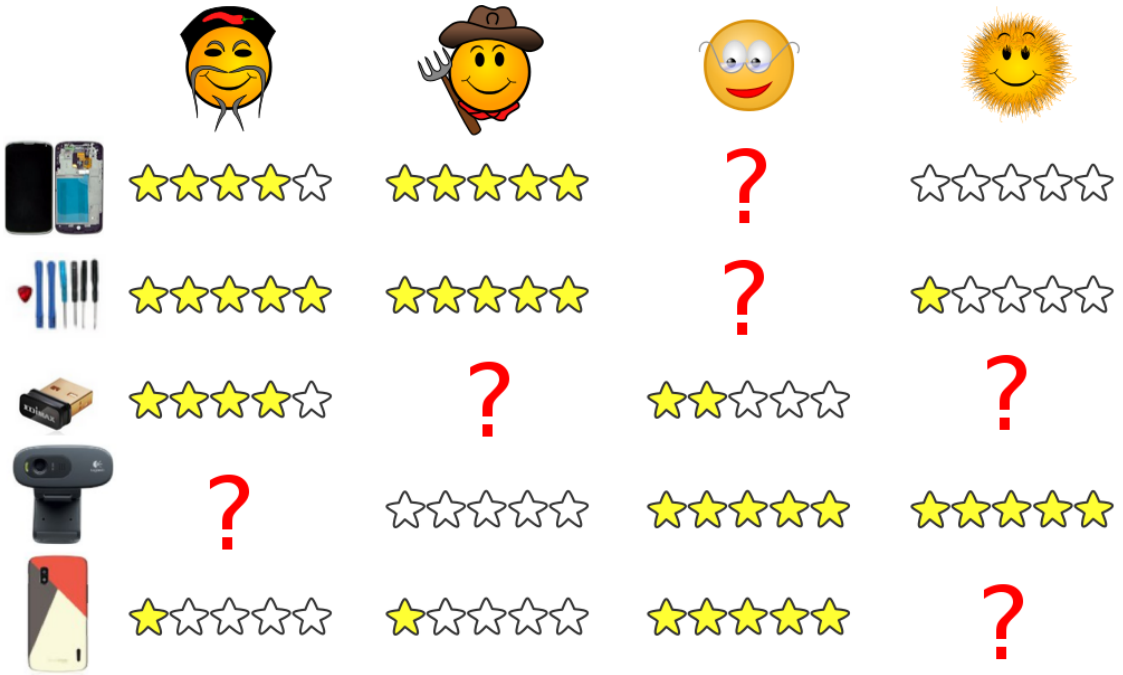
Recommender systems

Task/Problem of Recommender systems

Given these ratings for a number of products, predict how the user-ratings for products that have not yet been rated




tools
accessories
?
0.9
0.0
?
1.0
0.3
?
?
0.4
0.8
?
0.2
0.9
?
0.0
1.0


tools
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0.9
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1.0
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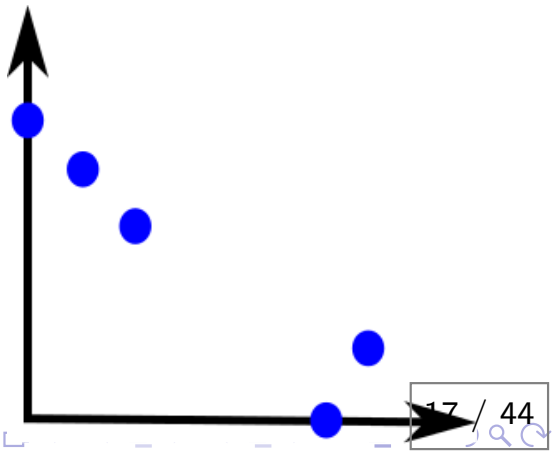
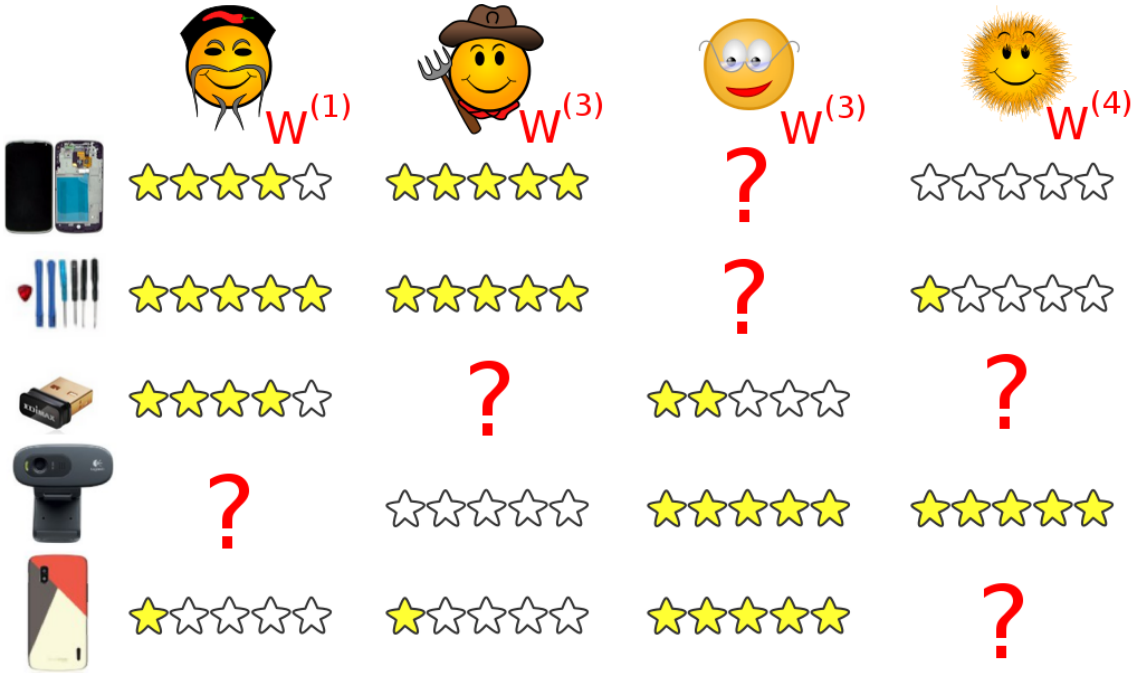
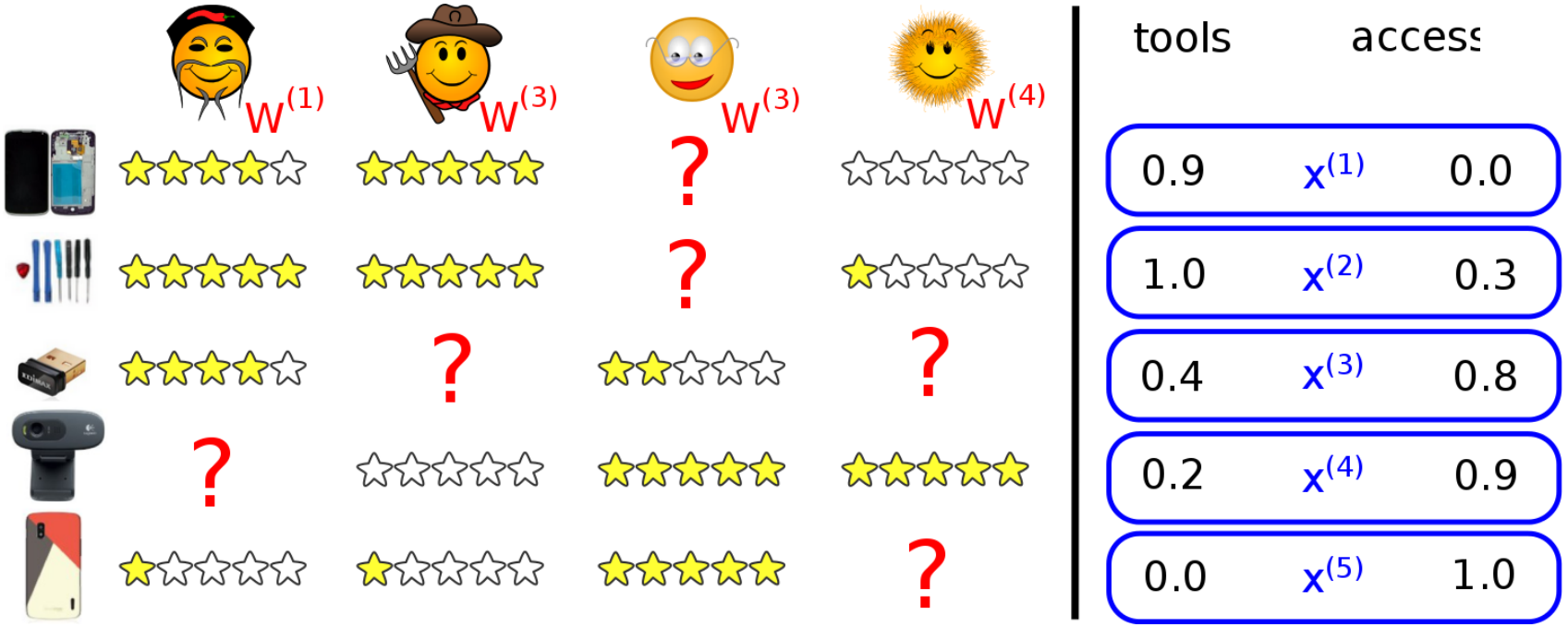




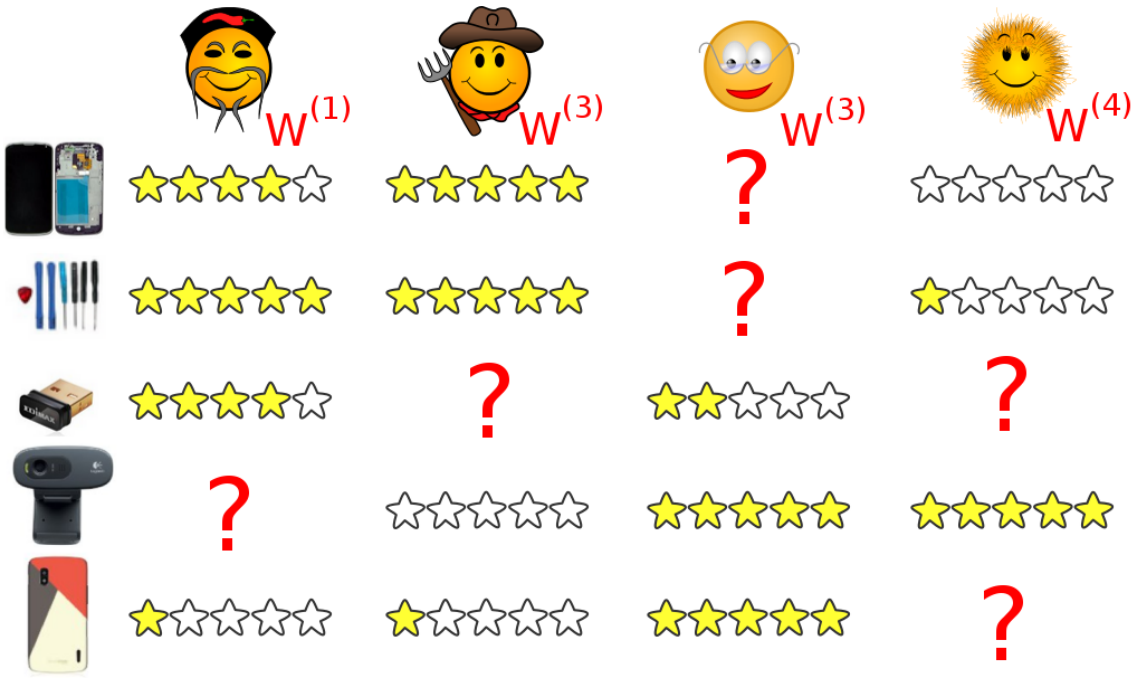
tools		accessories
0.9	$x^{(1)}$	0.0
1.0	$x^{(2)}$	0.3
0.4	$x^{(3)}$	0.8
0.2	$x^{(4)}$	0.9
0.0	$x^{(5)}$	1.0



Represent items as feature vectors

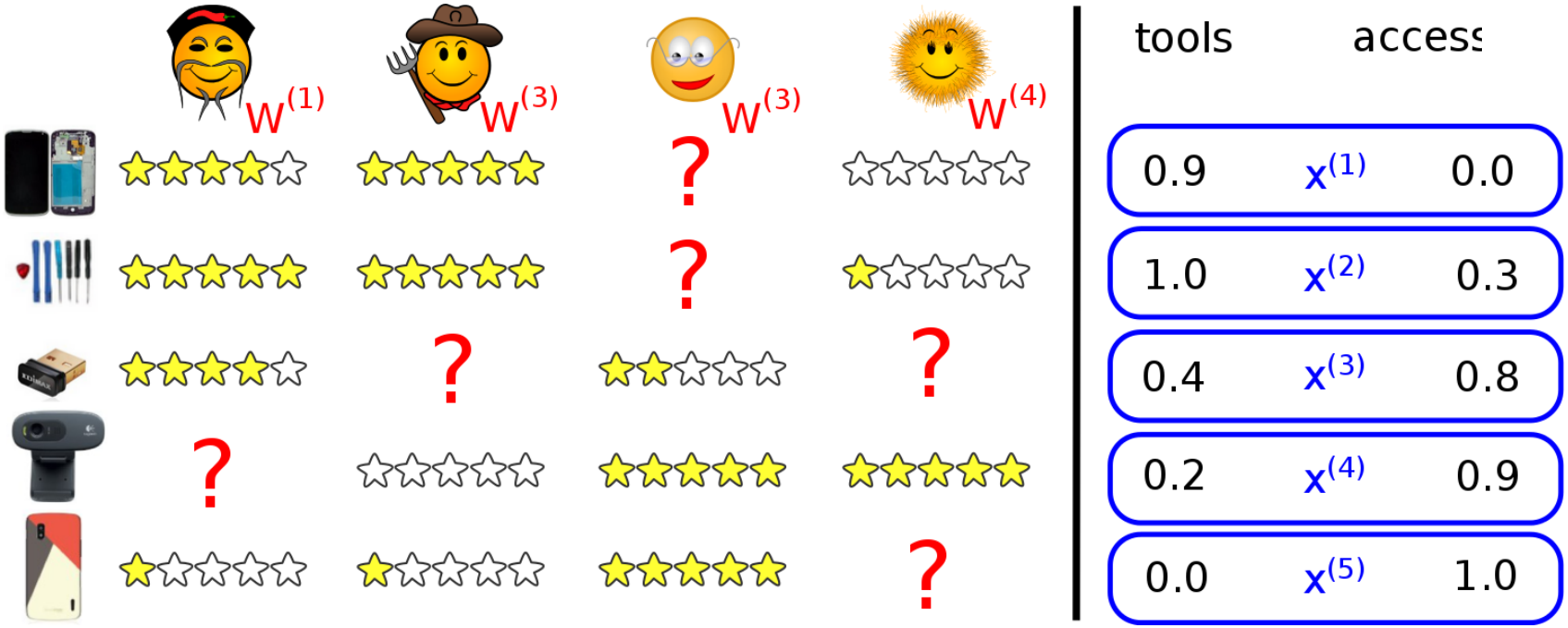
$$x^{(4)} = \begin{bmatrix} 0.2 \\ 0.9 \end{bmatrix} \quad W^{(1)} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\Rightarrow \left(W^{(1)}\right)^T x^{(4)} = 0.2 \cdot 5 + 0.9 \cdot 1 = 1,9$$



Learn weights from provided ratings for single user j (Linear regression):

$$\min_{W^{(j)}} \frac{1}{2} \sum_{i:y^{(i,j)}\neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right) + \frac{\lambda}{2} \sum_{k=1}^{F} \left(W_k^{(j)} \right)^2$$



Learn weights from provided ratings for single all users $1, \ldots, N$:

$$\min_{W^{(1)},\ldots,W^{(N)}} \frac{1}{2} \sum_{j=1}^{N} \sum_{i:y^{(i,j)}\neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right) + \frac{\lambda}{2} \sum_{j=1}^{N} \sum_{k=1}^{F} \left(W_k^{(j)} \right)^2$$

Recommender systems

Optimisation algorithm

$$\min_{W^{(1)},\ldots,W^{(N)}} \frac{1}{2} \sum_{j=1}^{N} \sum_{i:y^{(i,j)}\neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right) + \frac{\lambda}{2} \sum_{j=1}^{N} \sum_{k=1}^{F} \left(W_k^{(j)} \right)^2$$

Recommender systems

Optimisation algorithm

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Gradient descent update:

$$W_k^{(j)} = W_k^{(j)} - \alpha \left(\sum_{i=y^{(i,j)}\neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)} + \lambda W_k^{(j)} \right)$$

Recommender systems

Optimisation algorithm

$$\min_{W^{(1)},\ldots,W^{(N)}} \frac{1}{2} \sum_{j=1}^{N} \sum_{i:y^{(i,j)}\neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right) + \frac{\lambda}{2} \sum_{j=1}^{N} \sum_{k=1}^{F} \left(W_k^{(j)} \right)^2$$

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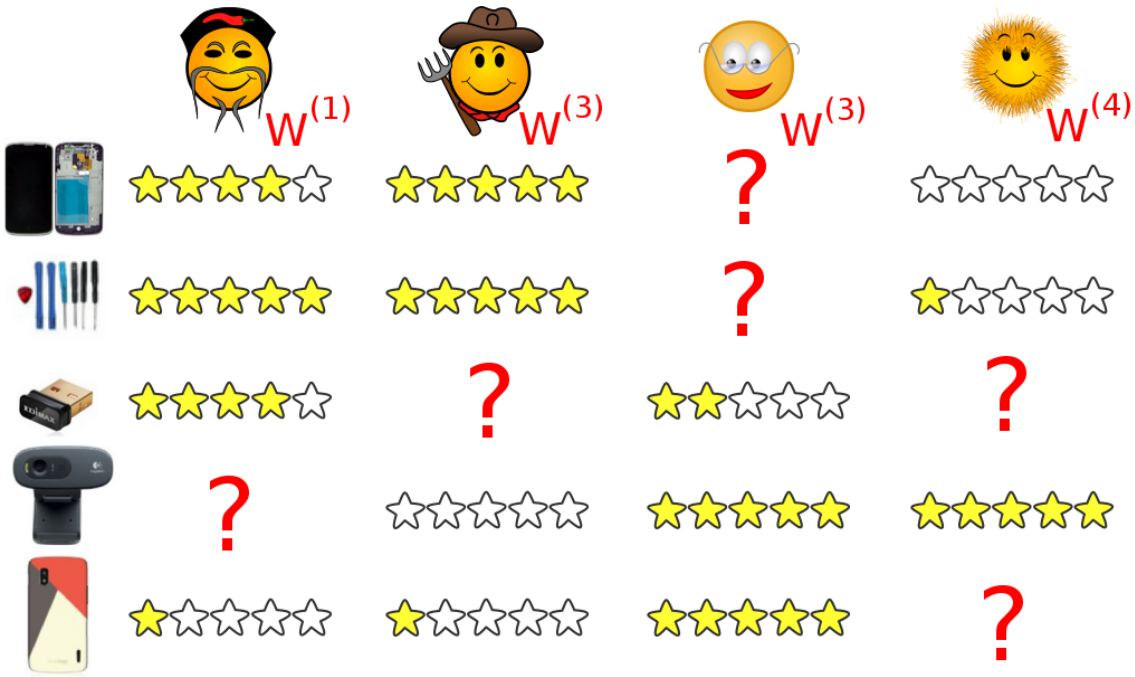
$$W_k^{(j)} = W_k^{(j)} - \alpha \left(\underbrace{\sum_{i=y^{(i,j)}\neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)} + \lambda W_k^{(j)}}_{\text{partial derivative}} \right)$$

Recommender systems

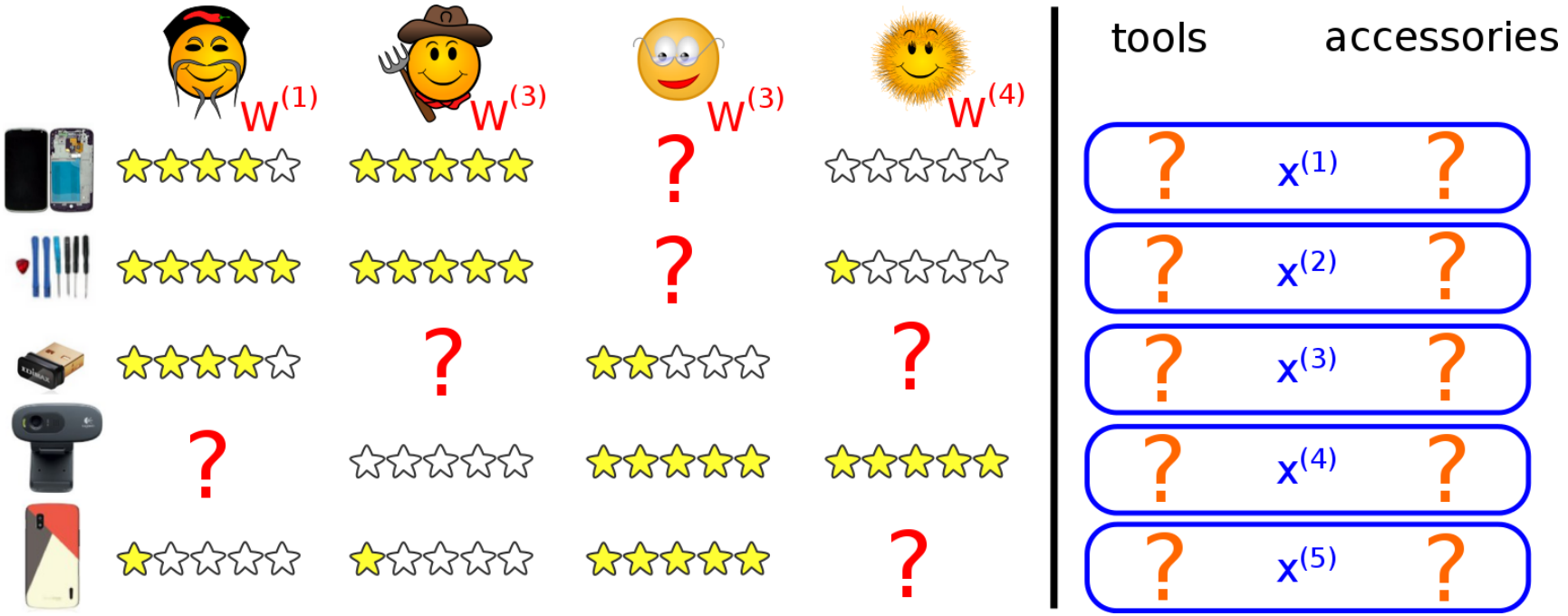
Collaborative filtering



We are able to calculate the weights given the feature vectors

Recommender systems

Collaborative filtering



We are able to calculate the weights given the feature vectors

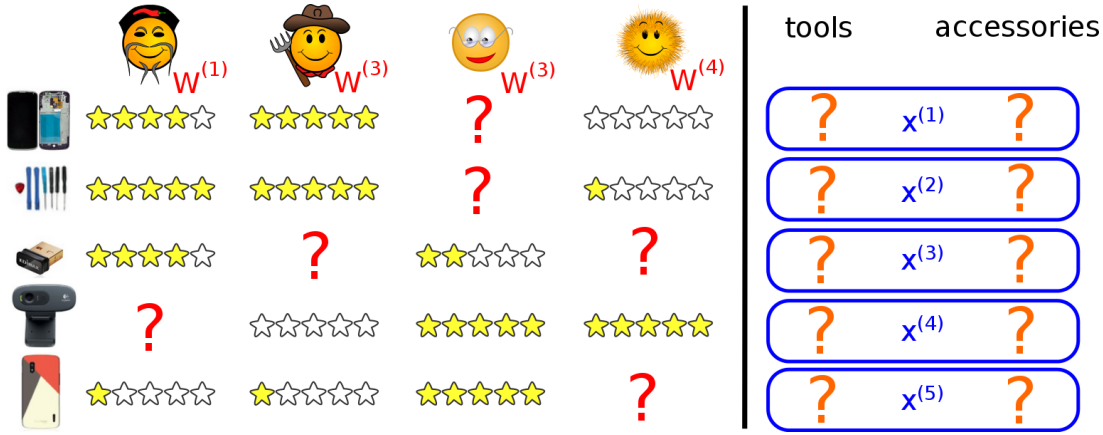
$\rightarrow$ But how do we obtain these feature vectors?

Recommender systems

Collaborative filtering

Users might tell their preferences

e.g. more interested in tools or accessories



$$W^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad W^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad W^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad W^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Recommender systems

Collaborative filtering

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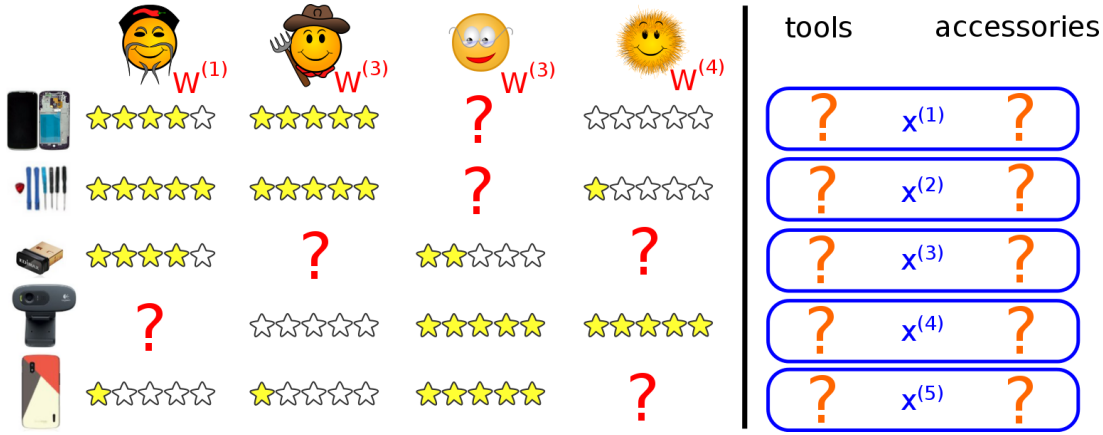


$$W^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad W^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad W^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad W^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\left(W^{(1)}\right)^T x^{(1)} \approx 4; \quad \left(W^{(2)}\right)^T x^{(1)} \approx 5;$$

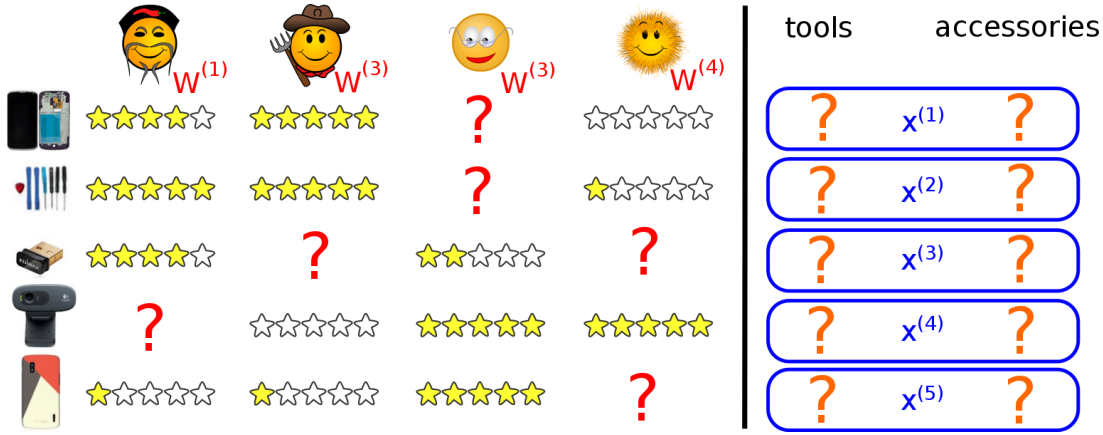
$$\left(W^{(3)}\right)^T x^{(1)} \approx ?; \quad \left(W^{(4)}\right)^T x^{(1)} \approx 0$$

Recommender systems

Collaborative filtering

Users might tell their preferences

e.g. more interested in tools or accessories



$$W^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad W^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad W^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad W^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\left(W^{(1)}\right)^T x^{(1)} \approx 4; \quad \left(W^{(2)}\right)^T x^{(1)} \approx 5;$$

$$\left(W^{(3)}\right)^T x^{(1)} \approx ?; \quad \left(W^{(4)}\right)^T x^{(1)} \approx 0$$

From these weights we can estimate the feature values

Recommender systems

Collaborative filtering

Optimisation algorithm

Given the weights/preferences $W^{(1)}, \ldots, W^{(N)}$, we are able to infer a feature $x^{(i)}$

$$\min_{x^{(i)}} \frac{1}{2} \sum_{j:y^{(i,j)} \neq ?} \left(\left(W^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{k=1}^{F} \left(x_k^{(i)} \right)^2$$

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Recommender systems

Collaborative filtering

Naive (iterative) Collaborative filtering algorithm

Given $x^{(1)}, \ldots, x^{(n)}$, we are able to estimate $W^{(1)}, \ldots, W^{(N)}$

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Collaborative filtering (naive)

Init: Randomly initialise the $W^{(i)}$

Repeat:
- Estimate the $x^{(i)}$ from the $W^{(i)}$
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Repeat:
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- Estimate the $W^{(i)}$ from the $x^{(i)}$

$\rightarrow$ CF iteratively improves the estimates for $x^{(i)}$ and $W^{(i)}$

$\rightarrow$ Algorithm collaborates with users: by providing some information about their preferences, it computes and improves the features

Recommender systems

Collaborative filtering

$$\min_{W^{(1)},\dots,W^{(N)}} \quad \frac{1}{2}\sum_{j=1}^{N}\sum_{i:y^{(i,j)}\neq ?}\left(\left(W^{(j)}\right)^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{N}\sum_{k=1}^{F}\left(W_k^{(j)}\right)^2$$

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Minimize $W^{(i)}$ and $x^{(i)}$ simultaneously:

$$\min_{x^{(1)},\dots,x^{(n)},W^{(1)},\dots,W^{(N)}} \quad \frac{1}{2}\sum_{i,j:y^{(i,j)}\neq ?}\left(\left(W^{(j)}\right)^T x^{(i)} - y^{(i,j)}\right)^2$$
$$+\frac{\lambda}{2}\sum_{i=1}^{n}\sum_{k=1}^{F}\left(x_k^{(i)}\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{N}\sum_{k=1}^{F}\left(W_k^{(j)}\right)^2$$

Recommender systems

Collaborative filtering

Collaborative filtering algorithm

Collaborative filtering

Init: Randomly initialise the $W^{(j)}$ and $x^{(i)}$

Optimisation: Simultaneously minimise the above function for $W^{(j)}$ and $x^{(i)}$

Gradient descent:

$$x_k^{(i)} = x_k^{(i)} - \alpha \left(\sum_{j=y^{(i,j)} \neq ?} \left(\left(W^{(j)}\right)^T x^{(i)} - y^{(i,j)} \right) W_k^{(j)} + \lambda x_k^{(i)} \right)$$

$$W_k^{(j)} = W_k^{(j)} - \alpha \left(\sum_{i=y^{(i,j)} \neq ?} \left(\left(W^{(j)}\right)^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)} + \lambda W_k^{(j)} \right)$$

Prediction: For a user i with parameters $W^{(j)}$ and an item with learned featues x, estimate a rating of $\left(W^{(j)}\right)^T x$

Outline

Stochastic Classification

Classification algorithm typically become very slow when data size increases

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→ This is because they loop repeatedly over the complete data set until convergence

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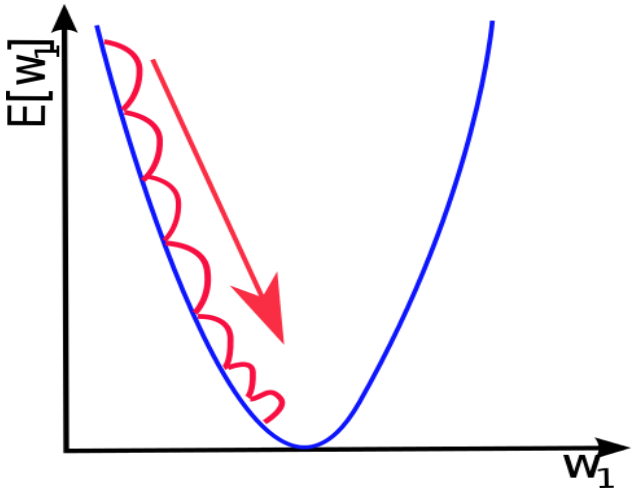
Solution

- Randomly iterate the update only over individual items instead of repeatedly considering the whole data set.

Stochastic Classification

Example: Gradient descent



$$\text{minimize } E[W] = \frac{1}{2n}\sum_{i=1}^{n}\left(W^T x^{(i)} - y^{(i)}\right)^2$$

$$\text{Repeat}\forall j : W_j = W_j - \delta \cdot \frac{\partial}{\partial W_j} E[W_j]$$

$$\rightarrow \forall j : W_j = W_j - \delta \cdot \frac{1}{n}\sum_{i=1}^{n}\left(W_j x_j^{(i)} - y^{(i)}\right) \cdot x_j^{(i)}$$

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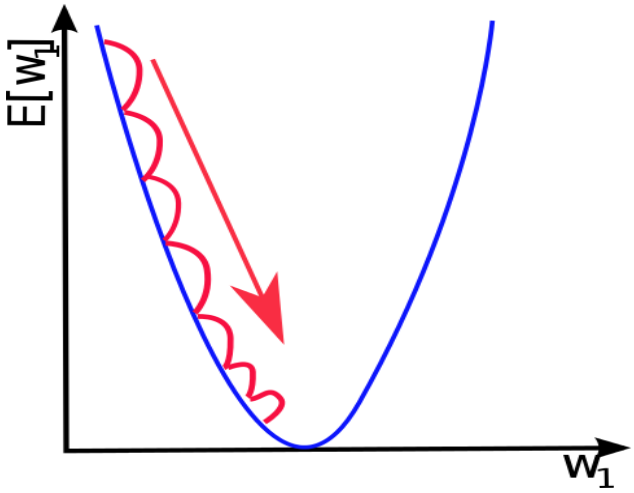

→ For single gradient descent-step, algorithms loops over all samples!

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Stochastic Classification

Example: Gradient descent

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\rightarrow \forall j : W_j &= W_j - \delta \cdot \frac{1}{n}\sum_{i=1}^{n}\left(W_j x_j^{(i)} - y^{(i)}\right) \cdot x_j^{(i)}
\end{aligned}$$

Stochastic Classification

Example: Gradient descent

→ For a single gradient descent-step, the algorithms loops over all samples!

To speed up the algorithm, compute gradient descent updates from individual training samples (randomly ordered)

$$\text{minimize } E[W] = \frac{1}{2n}\sum_{i=1}^{n}\left(W^T x^{(i)} - y^{(i)}\right)^2$$

$$\text{Repeat}\forall j : W_j = W_j - \delta \cdot \frac{\partial}{\partial W_j} E[W_j]$$

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Stochastic Classification

Example: Gradient descent

Standard:

$$\begin{aligned} \text{minimize } E[W] &= \frac{1}{2n}\sum_{i=1}^{n}\left(W^T x^{(i)} - y^{(i)}\right)^2 \\ \text{Repeat}\forall j : W_j &= W_j - \delta \cdot \frac{\partial}{\partial W_j} E[W_j] \\ \rightarrow \forall j : W_j &= W_j - \delta \cdot \frac{1}{n}\sum_{i=1}^{n}\left(W_j x_j^{(i)} - y^{(i)}\right) \cdot x_j^{(i)} \end{aligned}$$

Stochastic Classification

Example: Gradient descent

Standard:

$$\text{minimize } E[W] = \frac{1}{2n}\sum_{i=1}^{n}\left(W^T x^{(i)} - y^{(i)}\right)^2$$

$$\text{Repeat} \forall j : W_j = W_j - \delta \cdot \frac{\partial}{\partial W_j} E[W_j]$$

$$\rightarrow \forall j : W_j = W_j - \delta \cdot \frac{1}{n}\sum_{i=1}^{n}\left(W_j x_j^{(i)} - y^{(i)}\right) \cdot x_j^{(i)}$$

Stochastic:

Repeat over all training examples i (random order):

$$\Rightarrow \forall j : \quad W_j = W_j - \delta \cdot \left(W_j x_j^{(i)} - y^{(i)}\right) \cdot x_j^{(i)}$$

Stochastic Classification

The stochastic implementation will generally move the parameters towards the global minimum (...but not always!)

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Tradeoff use $1 \leq k \leq n$ random examples for each gradient descent update

$I = 1, 1 + k, 1 + 2k, \ldots$

$$\Rightarrow \forall j : W_j = W_j - \delta \cdot \frac{1}{I} \sum_{I=i}^{i+k} \left(W_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

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$k > 1$ might be faster than $k = 1$ for parallelized code

Outline

Online learning

Problem formulation

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Given a continuous stream of input data, update the parameters of your algorithm on-the-fly

Example: learn user behaviour from website users

Online learning

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Similar to stochastic classification: Update the parameters based on individual training examples

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Example: learn user behaviour from website users

Similar to stochastic classification: Update the parameters based on individual training examples

$$\Rightarrow \forall j: \quad W_j = W_j - \delta \cdot \left(h(x^{(i)}) - y^{(i)}\right) \cdot x_j^{(i)}$$

$\Rightarrow$ Able to adapt to changing user behaviour over time

Outline

Questions?

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stephan.sigg@cs.uni-goettingen.de

Literature

- C.M. Bishop: Pattern recognition and machine learning, Springer, 2007.
- R.O. Duda, P.E. Hart, D.G. Stork: Pattern Classification, Wiley, 2001.