Advanced Computer Networks
SS 2016

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Outline of Wireless Block

- Game theory and its applications
  - Game theory basics and concepts
  - Distributed Spectrum Sharing Application
- Social Group Maximization Framework
  - Introduction to the framework
  - Wireless Network Applications
- Mobile Data Offloading
  - Basics and ideas
  - Optimized Offloading Decision

Final Exam in this block only covers basic concepts and examples

Introduction to Game Theory

Game Theory

Rational – user aims to optimize its own objective

Interaction – user needs to take others' decisions into account

"…Game Theory is designed to address situations in which the outcome of a person's decision depends not just on how they choose among several options, but also on the choices made by the people they are interacting with…" --David Easley and Jon Kleinberg

"… Game theory is the study of the ways in which *strategic interactions* among *rational agents* produce *outcomes* with respect to the *utilities* of those agents …." --Stanford Encyclopedia of Philosophy

A Brief History

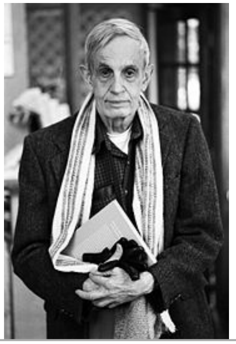
- 1944: Von Neumann and Oskar Morgenstern
  Theory of Games and Economic Behavior
  Two-player games

- 1950: John Nash
  Nash Equilibrium
  Equilibrium points in n-player games

- After 1950s: widely used in economics, politics, biology…
  Competition between firms
  Auction design
  Role of punishment in law enforcement
  International policies
  Evolution of species

O. Morgenstern 1902-1977

von Neumann 1903-1957

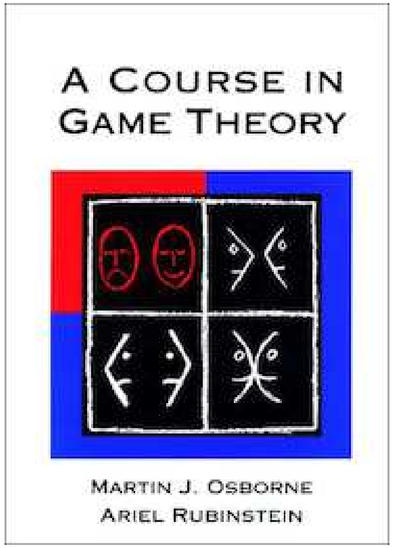


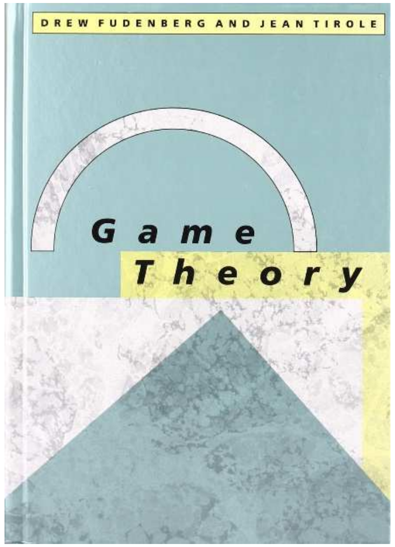
John Nash 1928-2015

Relevance to Networking Research

- Economic issues become increasingly important
  - Interactions with/between human users
    e.g., data plan pricing, resource allocation
  - Independent service providers
    e.g., bandwidth trading, peering agreements

- Tool for system design
  - Distributed algorithms
  - Multi-objective optimization
  - Incentive compatible protocols





Game Theory Basics

- Strategic game form (P, S, U)
  - Players $(P_1, \dots, P_N)$: finite number of decision makers
  - Strategy sets $(S_1, \dots, S_N)$: player P_i has a nonempty set S_i of actions/strategies s_i
  - Payoff function $U_i(s_1, \dots, s_N)$: player's preference/individual utility

- Pure Nash equilibrium (NE)
  - A strategy profile $(s_1^*, \dots, s_i^*, \dots, s_N^*)$ is a NE if for each player i

$$U_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \geq U_i(s_1^*, \dots, s_i, \dots, s_N^*), \forall s_i \in S_i$$

  - No player has incentive to deviate (stable system point)
  - NE is a fixed point of the best response functions

$$s_i^* = \underset{s_i \in S_i}{\operatorname{argmax}}\, U_i(s_1^*, \dots, s_i, \dots, s_N^*), \forall i$$

- There is no universal rule for finding a Nash equilibrium!

Prisoner's Dilemma

- Two suspects are arrested
- The police lack sufficient evidence to convict the suspects, unless at least one confesses
- The police hold the suspects in two separate rooms, and tell each of them three possible consequences:
  - If both deny: 1 month in jail each
  - If both confess: 6 months in jail each
  - If one confesses and one denies:
    - The one confesses: walk away free of charge
    - The one denies: serve 12 months in jail

Prisoner's Dilemma

strategies

Player 2

		Deny	Confess
Player 1	Deny	$-1, -1$	$-12, 0$
	Confess	$0, -12$	$-6, -6$

payoffs

Prisoner's Dilemma

- Strictly dominated strategy

– Player i's strategy s_i' is strictly dominated by player i's strategy s_i if

$$U_i(s_i, s_{-i}) > U_i(s_i', s_{-i}), \forall s_{-i}$$

where s_{-i} is the strategy profile of all the other players except player i

– No matter what other people do, by choosing s_i instead of s_i', player i will always obtain a better payoff

– Key principle: Never play a strictly dominated strategy

Prisoner's Dilemma

Player 2

Player 1		Deny	Confess
	Deny	$-1, -1$	$-12, 0$
	Confess	$0, -12$	$-6, -6$

Player 2's choice: Confess

Player 1's choice: Confess

NE of the game: $-6, -6$

Deny is strictly dominated by Confess!

Finding Nash Equilibrium

- When there are no strictly dominated strategies, we can not easily “simplify” the game
- Nash equilibrium is a state of mutual best responses
- Key principle: derive the best responses

Stag Hunt

- Two hunters decide what to hunt independently
- Each one can hunt a stag (deer) or a hare
- Successful hunt of stag requires cooperation
- Successful hunt of hare can be done individually
- Simultaneous decisions without prior communications

Stag Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	5 , 5	0 , 2
	Hare	2 , 0	2 , 2

There is no strictly dominated strategy

Find out a player’s best response given the other player’s choice

Stag Hunt

Given Player 2 chooses Stag

Player 1's best response

Player 1		Player 2: Stag	Player 2: Hare
	Stag	*5* , *5*	*0* , *2*
	Hare	*2* , *0*	*2* , *2*

Stag Hunt

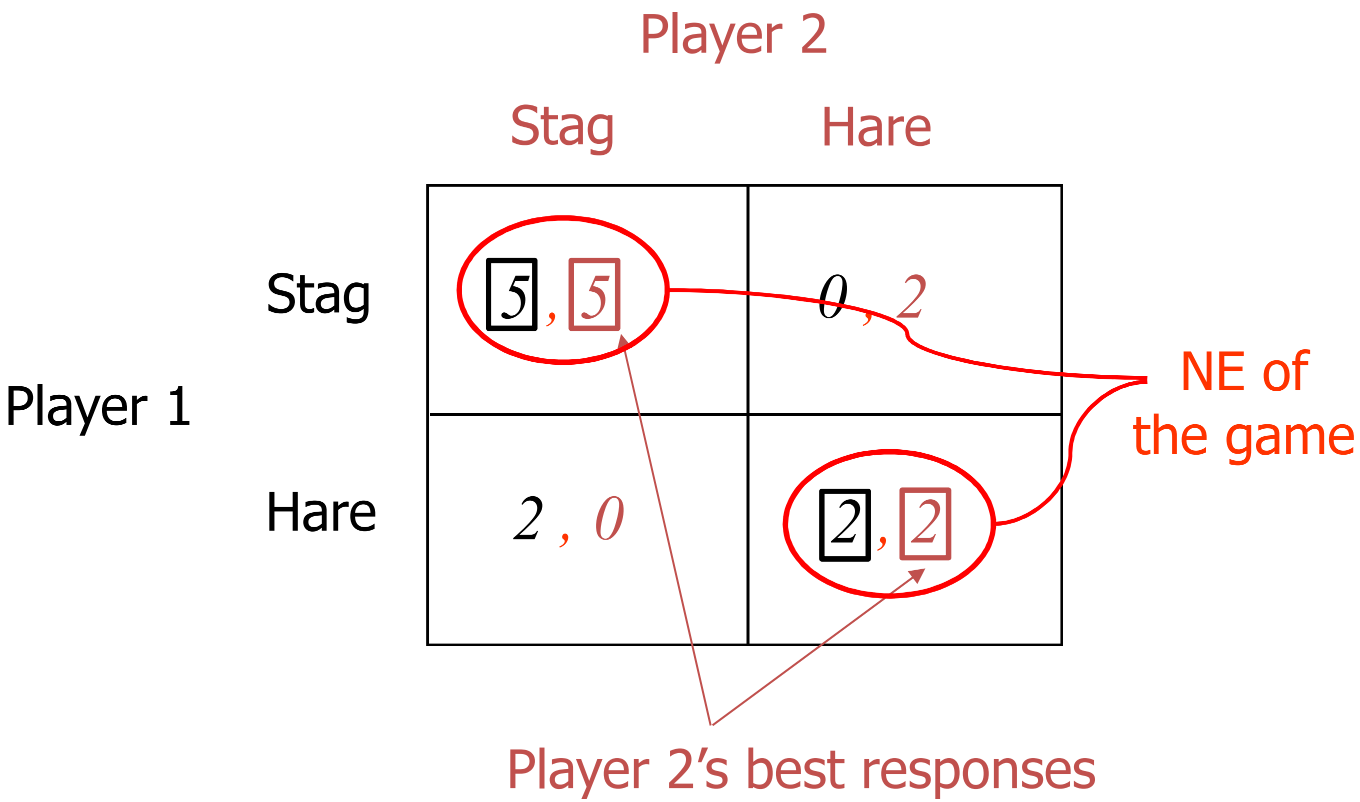
Given Player 2 chooses Hare

Player 2

Player 1	Stag	Hare
Stag	[5], 5	0, 2
Hare	2, 0	[2], 2

Player 1's best response → Hare

Stag Hunt



NE is a state of mutual best responses

Stag Hunt

- Two Nash equilibria exist

- (Stag, Stag) is payoff dominant
  - Both players get the best payoff possible
  - Require trust among players to achieve coordination

- (Hare, Hare) is risk dominant
  - Minimum risk if player is uncertain of each other's choice

Battle of Sexes

- A couple decide where to go during Friday night without communications
- Husband prefers to go and watch football
- Wife prefers to go and watch ballet
- Both prefer to stay together during the night

Battle of Sexes

		Wife: Football	Wife: Ballet
Husband	Football	4 , 2	0 , 2
	Ballet	0 , 0	2 , 4

There is no strictly dominated strategy

Find out a player's best response given the other player's choice

Battle of Sexes

Given Wife chooses Football

Husband's best response

		Wife	
		Football	Ballet
Husband	Football	[4], 2	0 , 0
	Ballet	0, 0	2 , 4

Battle of Sexes

Given Wife chooses Football

Wife

Husband		Football	Ballet
	Football	4, 2	0 , 0
	Ballet	0, 0	[2], 4

Husband's best response

Battle of Sexes

		Wife: Football	Wife: Ballet
Husband	Football	*4*, *2*	*0*, *0*
	Ballet	*0*, *0*	*2*, *4*

NE of the game

Wife's best responses

NE is a state of mutual best responses

Cournot Competition

- Prisoner's dilemma, Stag Hunt, Battle of Sexes are finite games with finite number of actions for each player
- Cournot competition is a continuous game in which a player has continuous (infinite) choices

Cournot Competition

- Two firms producing the same kind of product in quantities of q_1 and q_2, respectively
- Market clearing price $p = A - q_1 - q_2$
- Cost of unit production is C for both firms
- Profit for firm i

$$\begin{aligned} J_i &= (p - C)q_i \\ &= (A - C - q_1 - q_2)q_i \end{aligned}$$

Define $B \equiv A - C$

- Objective of firm i: choose q_i to maximize profit

$$q_i^* = \underset{q_i}{\operatorname{argmax}} \ (B - q_1 - q_2)q_i$$

Cournot Competition

- Firm i's best response, given its competitor's q_j

$$q_i^* = (B - q_j)/2$$

- NE (q_1^*, q_2^*) of Cournot competition satisfies

$$\begin{cases} q_1^* = (B - q_2^*)/2 \\ q_2^* = (B - q_1^*)/2 \end{cases}$$

- This leads to the NE as

$$q_1^* = q_2^* = B/3$$

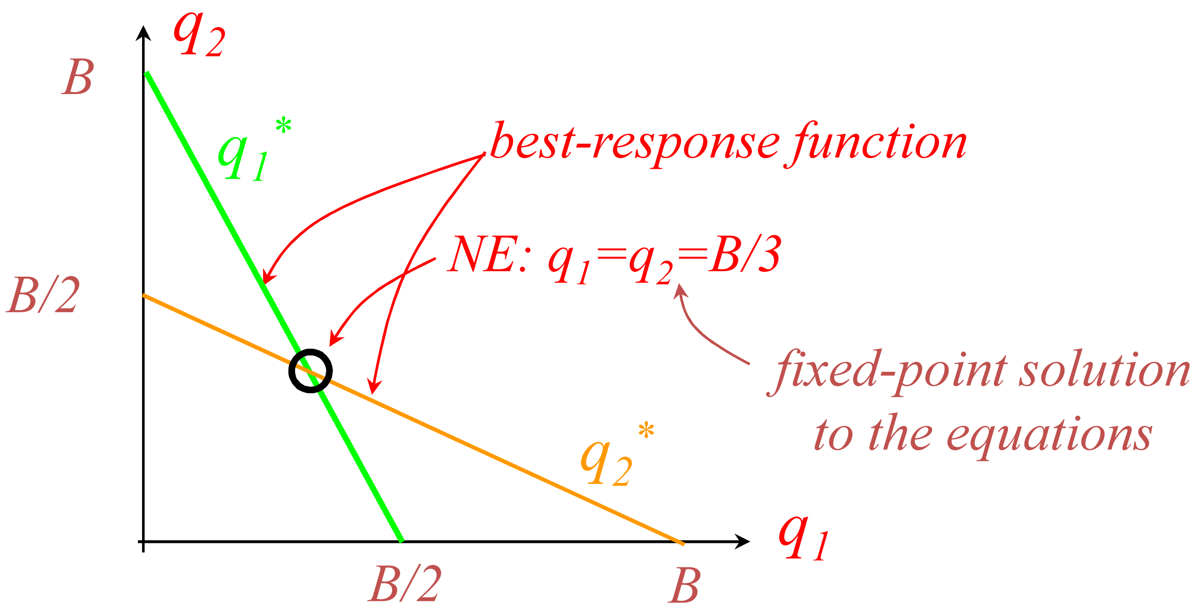
Cournot Competition

- Firm i's best response, given its competitor's q_j

$$q_i^* = (B - q_j)/2$$

- NE (q_1^*, q_2^*) of Cournot competition satisfies

$$\begin{cases} q_1^* = (B - q_2^*)/2 \\ q_2^* = (B - q_1^*)/2 \end{cases}$$



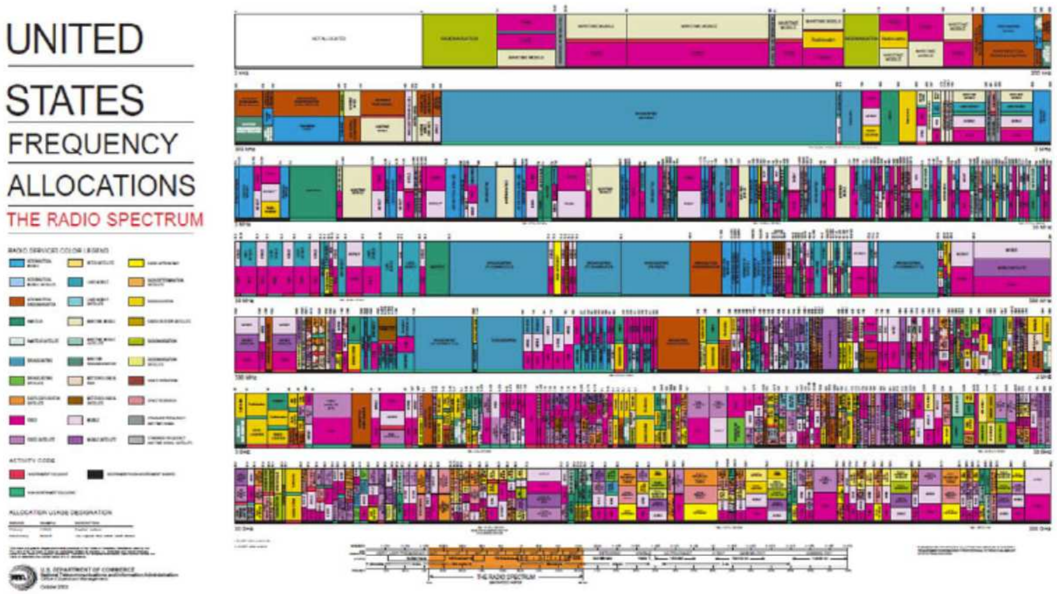
Summary

- Game theory: players, strategies, and payoffs
- Nash equilibrium
- Strictly dominated strategy
- Best response strategy
- Finite and infinite(continuous) games

Application in
Distributed Spectrum Sharing

Distributed Spectrum Sharing

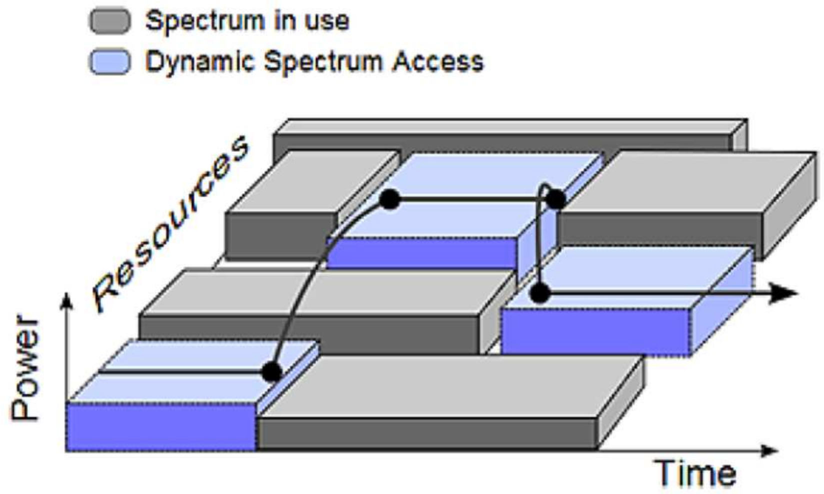
- Spectrum is scarce
  - Most spectrums have been exclusively licensed
  - More and more wireless devices emerge



- Spectrum is under-utilized
  - E.g., average spectrum utilization in Chicago is lower than 20%

Distributed Spectrum Sharing

- Dynamic spectrum access with cognitive radios
  - Address spectrum under-utilization problem
  - Primary user (PU) – licensed spectrum holder
  - Secondary user (SU) – unlicensed spectrum user
  - Enable SUs opportunistically share the spectrum with PUs



Distributed Spectrum Sharing

- Achieving efficient distributed spectrum sharing among SUs is challenging
  - Spectrum opportunities change over frequency, time, and space
  - Individual SU has limited observations of the entire network
  - Multiple simultaneous SUs may generate severe interference
- SUs are hence required to make intelligent decisions for efficient spectrum sharing

Distributed Spectrum Sharing

Key problem: how to share spectrum in an intelligent way?

- Individual intelligence
  - ▶ SUs share the spectrum competitively based on strategic interactions
  - ▶ SUs are fully rational (e.g., belong to different authorities)
  - ▶ Non-cooperative spectrum access games

Distributed Spectrum Sharing with Spatial Reuse

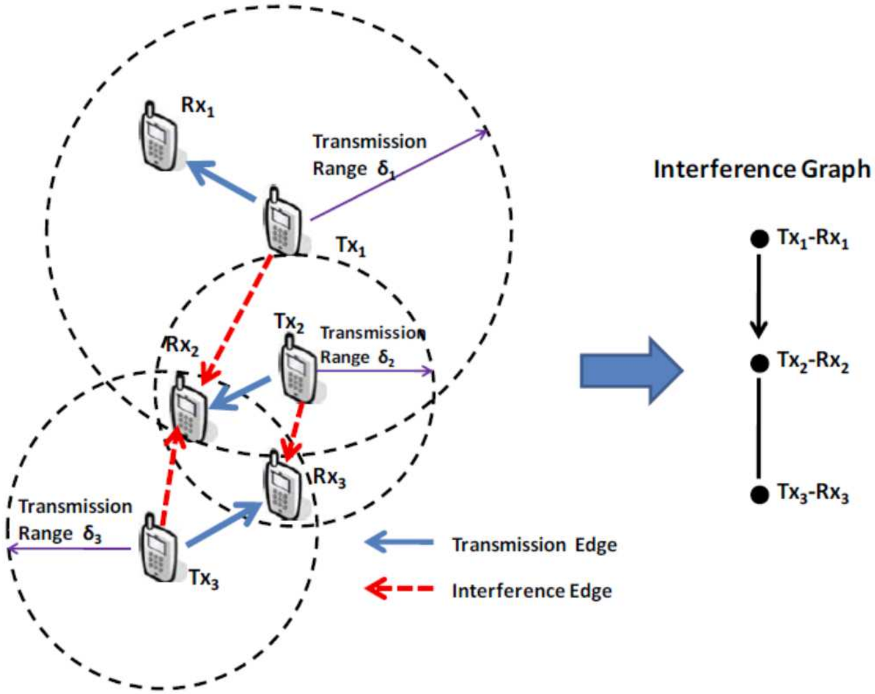
- SUs with individual intelligence share the spectrum based on competition
- Game theory for modeling competitive spectrum sharing
- Spatial aspect of competitive spectrum sharing is less understood
  - Spatial reuse is a key feature of wireless communication
  - Most existing works focus on fully interfering case
- Spatial spectrum access game for competitive spectrum sharing with spatial reuse

System Model

- M heterogeneous channels
  - Channel state $S_m(\tau) \in \{0, 1\}$ with an idle probability of θ_m
  - $\mathcal{M} = \{1, 2, ..., M\}$ – the set of channels
- N heterogeneous SUs
  - Data rate $b_m^n(\tau)$ follows i.i.d random process with mean B_m^n
  - $\mathcal{N} = \{1, 2..., N\}$ – the set of SUs

System Model

- Interference graph $G = \{\mathcal{N}, \mathcal{E}\}$
  - $\mathcal{N}$ – vertex set
  - $\mathbf{d}_n = (d_{Tx_n}, d_{Rx_n})$ – locations of transmitter and receiver of SU n
  - $\mathcal{E} = \{(i,j) : ||d_{Tx_i}, d_{Rx_j}|| \leq \delta_i, \forall i, j \neq i \in \mathcal{N}\}$ – interference edge set
  - $\mathcal{N}_n = \{i : (i,n) \in \mathcal{E}, i \in \mathcal{N}\}$ – interfering SUs of SU n



System Model

- Channel contention model
  - $\mathbf{a} = (a_1, ..., a_N)$ – channel selections of all SUs
  - $\mathcal{N}_n^{a_n}(\mathbf{a}) = \{i \in \mathcal{N}_n : a_i = a_n\}$ – interfering SUs choosing the same channel as SU n
  - $g_n(\mathcal{N}_n^{a_n}(\mathbf{a}))$ – probability that SU n can grab the channel
- Expected SU throughput

$$U_n(\mathbf{a}) = \theta_{a_n} B_{a_n}^n g_n(\mathcal{N}_n^{a_n}(\mathbf{a}))$$

Spatial Spectrum Access Game (SSAG)

- Each SU with individual intelligence tries to maximize its own throughput

$$\max_{a_n \in \mathcal{M}} U_n(a_n, a_{-n}), \forall n \in \mathcal{N}$$

- Spatial spectrum access game (SSAG) $\Gamma = (\mathcal{N}, \mathcal{M}, G, \{U_n\}_{n \in \mathcal{N}})$
- Key problems:
  - ▶ Does SSAG have a Nash equilibrium?
  - ▶ If SSAG has an equilibrium, how to achieve it?

Spatial Spectrum Access Game (SSAG)

- Key problems:
  - Does SSAG have a Nash equilibrium?
  - If SSAG has an equilibrium, how to achieve it?

Existence of Nash Equilibrium

- Pure Nash equilibrium $\mathbf{a}^* = (a_1^*, ..., a_N^*)$

$$a_n^* = \arg\max_{a_n} U_n(a_n, a_{-n}^*), \forall n \in \mathcal{N}$$

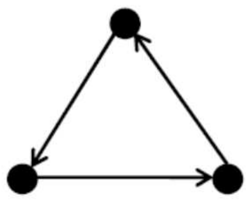
  - ▶ SUs are mutually satisfactory (no SU can improve unilaterally)
- Mixed strategy $\boldsymbol{\sigma}_n = (\sigma_1^n, ..., \sigma_M^n)$
  - ▶ σ_m^n – probability of SU n choosing channel m
  - ▶ $U_n(\boldsymbol{\sigma}_1, ..., \boldsymbol{\sigma}_N)$ – SU throughput under mixed strategies
- Mixed Nash equilibrium $(\boldsymbol{\sigma}_1^*, ..., \boldsymbol{\sigma}_N^*)$

$$\boldsymbol{\sigma}_n^* = \arg\max_{\boldsymbol{\sigma}_n} U_n(\boldsymbol{\sigma}_n, \boldsymbol{\sigma}_{-n}^*), \forall n \in \mathcal{N}$$

- Example: 2 SUs, 2 Channels, and $U_n(\mathbf{a}) = p \prod_{i \in \mathcal{N}_n^{a_n}(\mathbf{a})} (1-p)$
  - ▶ Pure Nash equilibrium: $(1, 2)$ and $(2, 1)$
  - ▶ Mixed Nash equilibrium: $((\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}))$

Existence of Nash Equilibrium

- SSAG always has a mixed Nash equilibrium
  - SSAG is a finite game
  - Finite game always has a mixed Nash equilibrium (John Nash 1950)
  - Require channel switching
- Pure Nash equilibrium requires no channel switching
- SSAG may not admit a pure Nash equilibrium
  - Example: 3 SUs, 2 channels, $U_n(\mathbf{a}) = p \prod_{i \in \mathcal{N}_n^{a_n}(\mathbf{a})} (1-p)$

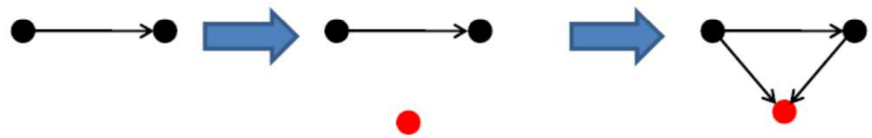


- Graphical structure plays a key role

Existence of Nash Equilibrium on Directed Graphs

Lemma one:

- Suppose that SSAG on a directed graph G has a pure Nash equilibrium
- Construct a new SSAG by adding a new SU
  - Can not generate interference to SUs in original SSAG
  - May receive interference from SUs in original SSAG
- New SSAG game also has a pure Nash equilibrium

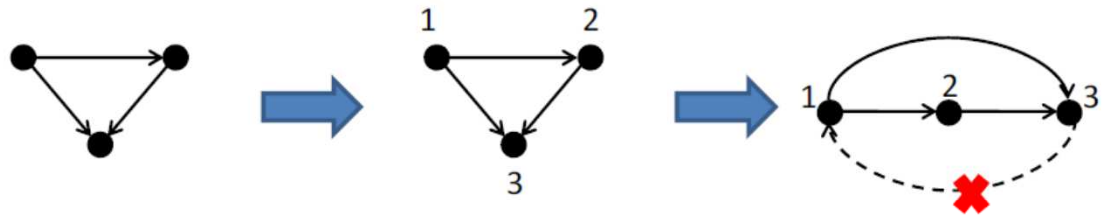


Existence of Nash Equilibrium on Directed Graphs

Theorem

SSAG on a directed acyclic graph has a pure Nash equilibrium.

- Directed acyclic graph can be given a topological sort (ordering of nodes)
- No edges directed from nodes of high order to nodes of low order



Existence of Nash Equilibrium on Directed Graphs

- Congestion property (CP):
  $\tilde{\mathcal{N}}_n^{a_n}(\mathbf{a}) \subseteq \mathcal{N}_n^{a_n}(\mathbf{a}) \Longrightarrow g_n(\tilde{\mathcal{N}}_n^{a_n}(\mathbf{a})) \geq g_n(\mathcal{N}_n^{a_n}(\mathbf{a}))$
  - More contending SUs, less chance to grab the channel
  - Natural for practical wireless systems

Existence of Nash Equilibrium on Directed Graphs

Lemma two:

- Suppose that SSAG satisfying CP on a directed graph G has a pure Nash equilibrium
- Construct a new SSAG by adding a new SU
  - ▶ Channel contention satisfies CP
  - ▶ May generate/receive interference to/from at most one SUs in original SSAG
- New SSAG game also has a pure Nash equilibrium

Existence of Nash Equilibrium on Directed Graphs

Lemma two:

- Suppose that SSAG satisfying CP on a directed graph G has a pure Nash equilibrium
- Construct a new SSAG by adding a new SU
  - ▶ Channel contention satisfies CP
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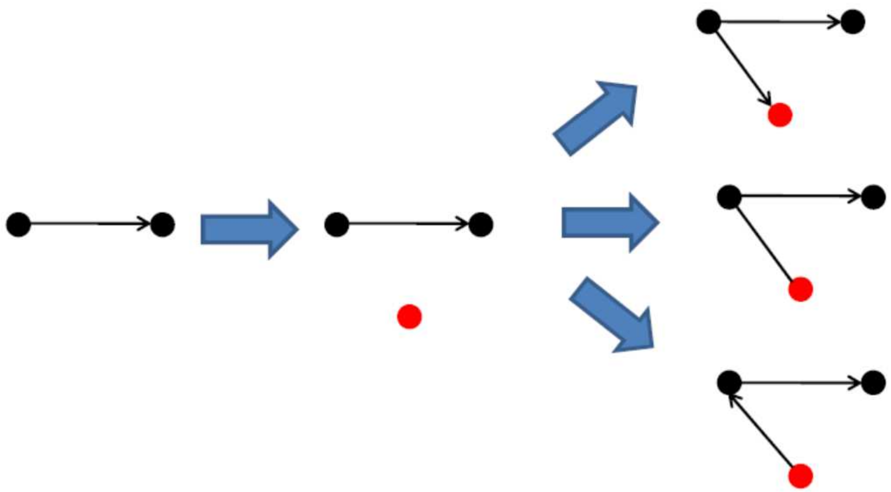


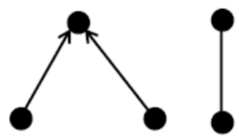
Existence of Nash Equilibrium on Directed Graphs

Theorem

SSAG satisfying CP on a directed tree/forest has a pure Nash equilibrium.

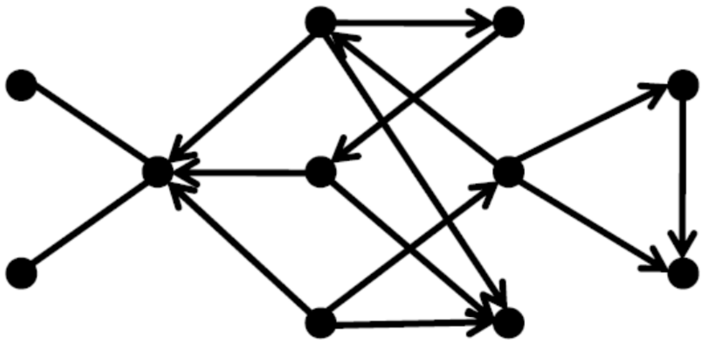
- Directed tree – the corresponding graph without directions is a tree

- Directed forest – a disjoint union of directed trees

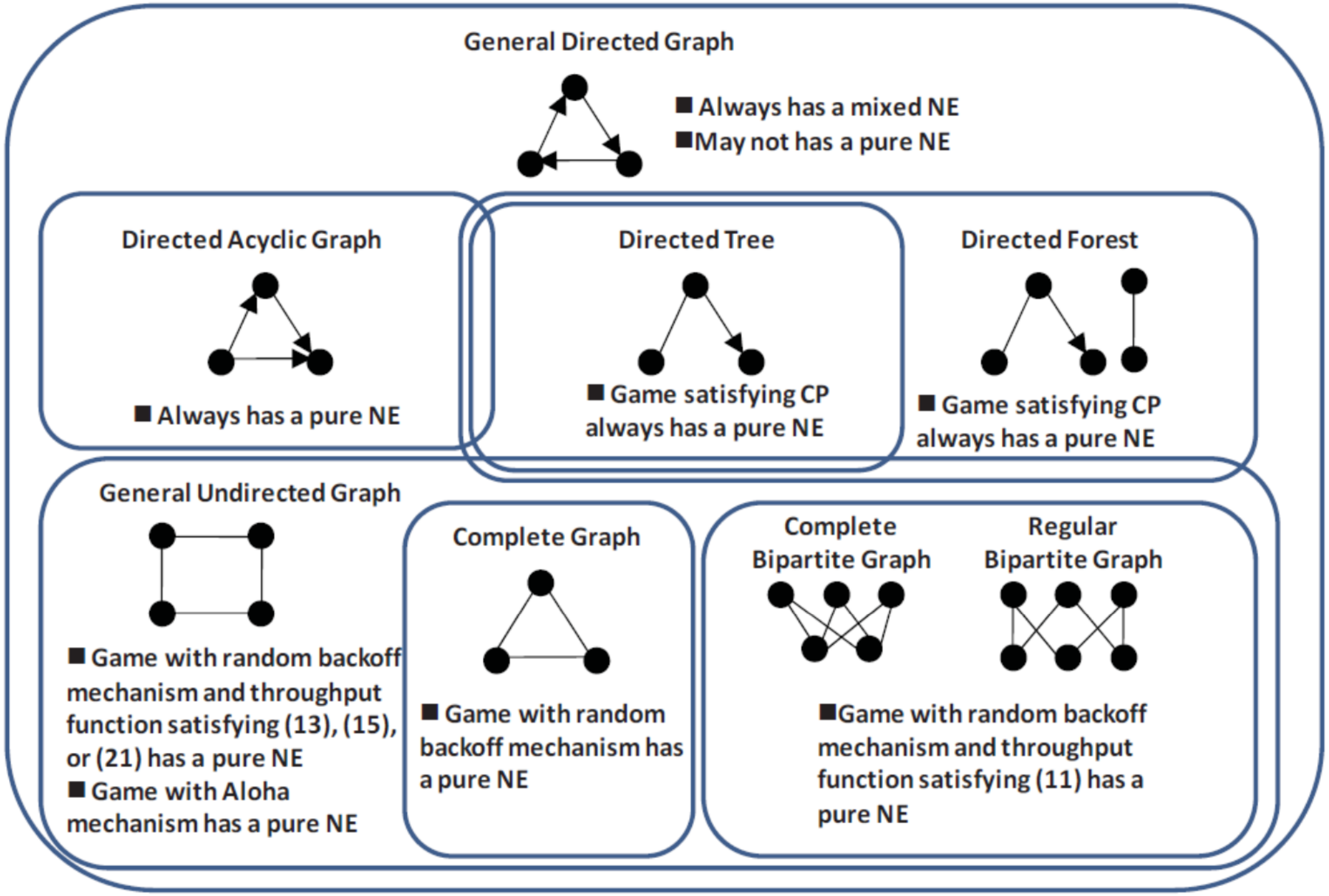


Existence of Nash Equilibrium on Directed Graphs

- More interference graphs can be constructed from Lemmas one & two



Existence of Nash Equilibrium



Spatial Spectrum Access Game (SSAG)

- Key problems:
  - Does SSAG have a Nash equilibrium?
  - If SSAG has an equilibrium, how to achieve it?

Achieving Equilibrium For SSAG

- SSAG always has a mixed Nash equilibrium
- Sharing information is not incentive compatible
  - SUs with individual intelligence are competitive
- How to achieve a mixed Nash equilibrium without information exchange?
- Distributed learning mechanism
  - Extend single-agent reinforcement learning to a multi-agent setting
  - Each SU estimates its throughput locally
  - Each SU adapts channel access strategy based on accumulated estimations

Distributed Learning For SSAG

- SU n chooses a channel $a_n(t)$ according to mixed strategy $\boldsymbol{\sigma}_n(t) = (\sigma_n^1(t), ..., \sigma_n^M(t))$
- Mixed strategy $\boldsymbol{\sigma}_n(t)$ is generated according to perceptions $\mathbf{P}_n(t) = (P_n^1(t), ..., P_n^M(t))$ as

$$\sigma_n^m(t) = \frac{\exp(\gamma P_n^m(t))}{\sum_{i=1}^M \exp(\gamma P_n^i(t))}$$

  - $P_n^m(t)$ – estimated performance of choosing channel m
  - γ – balance between exploration and exploitation
- Perceptions $\mathbf{P}_n(t)$ are updated as

$$P_n^m(t+1) = \begin{cases} (1-\mu_t)P_n^m(t) + \mu_t \tilde{U}_n(\mathbf{a}(t)), & \text{if } a_n(t) = m \\ P_n^m(t), & \text{otherwise} \end{cases}$$

  - μ_t – smoothing factor
  - Reinforce the perception of the channel just accessed

Convergence of Distributed Learning

- $\boldsymbol{\sigma} = (\boldsymbol{\sigma}_1, ..., \boldsymbol{\sigma}_N)$ is an ϵ-approximate mixed Nash equilibrium if

$$U_n(\boldsymbol{\sigma}_n, \boldsymbol{\sigma}_{-n}) \geq \max_{\hat{\boldsymbol{\sigma}}_n} U_n(\hat{\boldsymbol{\sigma}}_n, \boldsymbol{\sigma}_{-n}) - \epsilon, \forall n \in \mathcal{M}$$

  - ϵ – gap from an exact mixed Nash equilibrium

Convergence of Distributed Learning

Convergence by stochastic approximation theory

- γ satisfies

$$\gamma < \frac{1}{2\max_{m\in\mathcal{M},n\in\mathcal{N}}\{\theta_m B_m^n\}\max_{n\in\mathcal{N}}\{|\mathcal{N}_n|\}}$$

  ▶ Smaller $\gamma \Longrightarrow$ more random exploration $\Longrightarrow$ better environment understanding

- $\sum_t \mu_t = \infty$ and $\sum_t \mu_t^2 < \infty$

Theorem

Distributed learning for SSAG converges to an ϵ-approximate mixed Nash equilibrium $\boldsymbol{\sigma}^*$ with $\epsilon = \max_{n\in\mathcal{N}}\{-\frac{1}{\gamma}\sum_{m=1}^{M}\sigma_m^{n*}\ln\sigma_m^{n*}\}$.

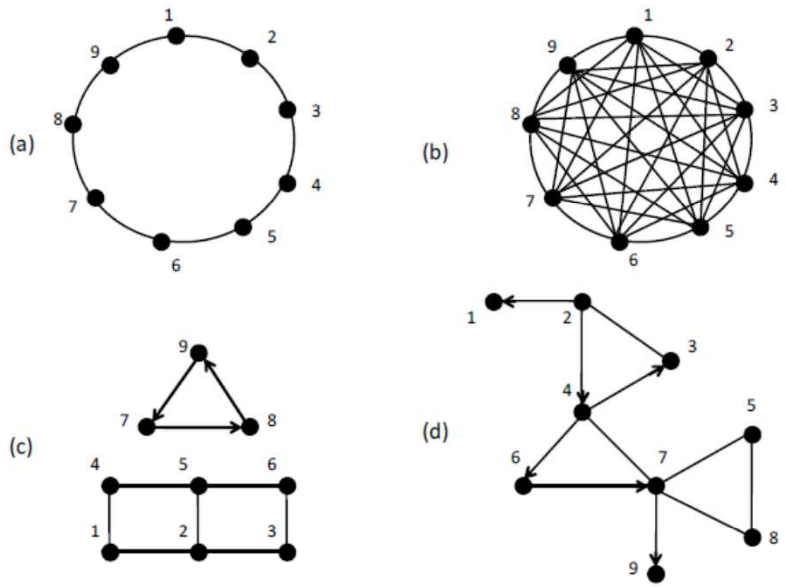
▶ $\epsilon \leq \frac{1}{\gamma}\ln M$, e.g., random channel access

▶ Convergence depends on the structure of interference graph

▶ Trade-off between exploration and exploitation: smaller γ for convergence $\Longrightarrow$ larger performance gab ϵ

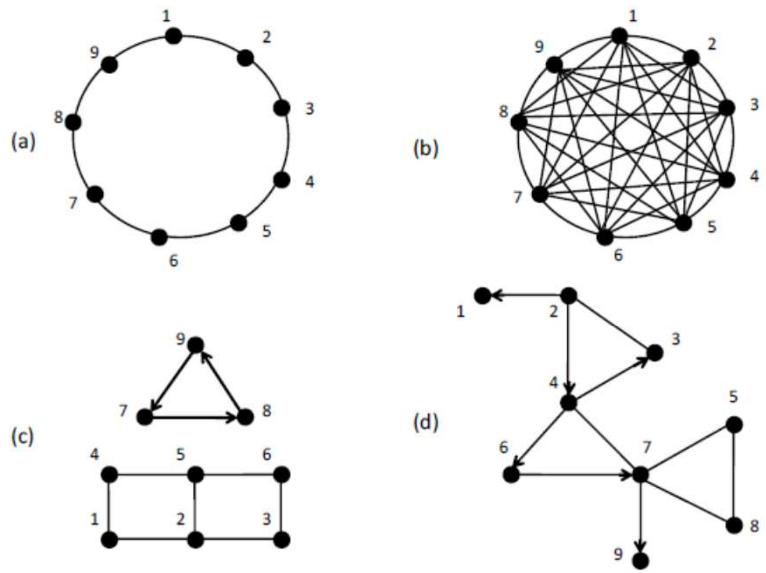
Simulation

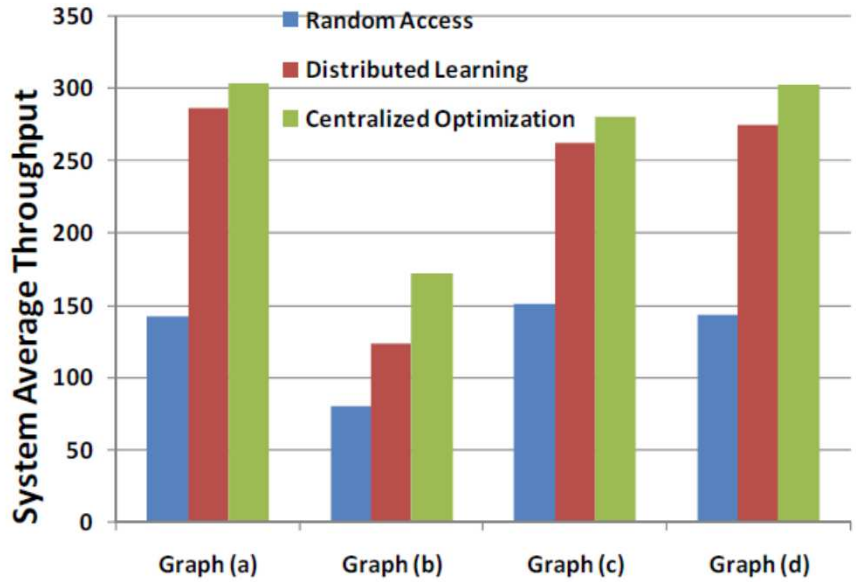
- $M = 5$ channels and $N = 9$ SUs on four different interference graphs



Simulation

- Up-to 100% performance improvement over random access





- At most 28% performance loss over centralized optimal solution on Graph (b) without spatial reuse
- At most 10% performance loss over centralized optimal solution on Graphs (a), (c), and (d) with spatial reuse

Summary of Spatial Spectrum Access Game

- SSAG for competitive spectrum sharing with spatial reuse
- Explore the existence of both pure and mixed Nash equilibrium
- Distributed learning for achieving approximate mixed Nash equilibrium